The three domes of the French Panthéon

Carlo Blasi[1], Eva Coïsson[2], Daniel Lefèvre[3]

Abstract: The French Panthéon was built in the late XVIII century with stone masonry reinforced with iron elements (pierre armée). The use of this innovative technique allowed the designers to adopt slender, unprecedented shapes, which were criticized by many at that time. The early appearance of cracks during its construction ignited a debate which is still a milestone in building history, as it marks the passage from the *Art* of Building to the *Science* of Building. More specifically, the debate focused on the thrust of the domes and on the consequent load eccentricity on the pillars.
Lately, due to the fall of stone pieces from the ceilings, the historical documents have been read and surveys, tests and numerical models have been carried out in order to identify the structural behaviour of this majestic building and its defects and thus to find the most compatible and respectful interventions to stop the damages.

Keywords: Reinforced stone masonry (pierre armée), historical documents, damage survey, structural analysis, thermal effects, conservation.

1. INTRODUCTION: TWO CENTURY OF DEBATES

The French Panthéon, designed by Soufflot and completed by Rondelet at the end of the XVIII century, has three superimposed domes, with an imposing outer colonnade, resting over a complex system of pillars and great arches (Figure 1). The inner dome, hemispherical and with a central hole, has only a scenographic purpose, the intermediate one, with a catenary shape, sustains the lantern, and the outer one completes the imposing external aesthetic impact of the monument. They are all made of stone masonry perfectly cut, with plaster bed joints only a few millimetres thick, reinforced with the widespread use of iron clamps (pierre armée). The outer dome, probably designed looking at the shape of the wooden dome of Saint Paul in London and to its colonnade, is very light, as it has the only functional purpose of protecting and covering the inner space, carrying its own weight. Externally, it is covered by a layer of lead.
The whole structure was much more slender than the traditional ones, thanks to the use of the new technique of the reinforced stone masonry, and did not follow the classical building rules.
The two inner domes are about 50 cm thick, while the outer dome is only 25 cm over a diameter of 28 m, with some larger stiffening ribs (50 cm thick): the comparison with other domes in the past, like Santa Maria del Fiore in Florence (4 m thick and 45 m wide) or San Pietro in Rome (5 m thick and 43 m wide) is astonishing.

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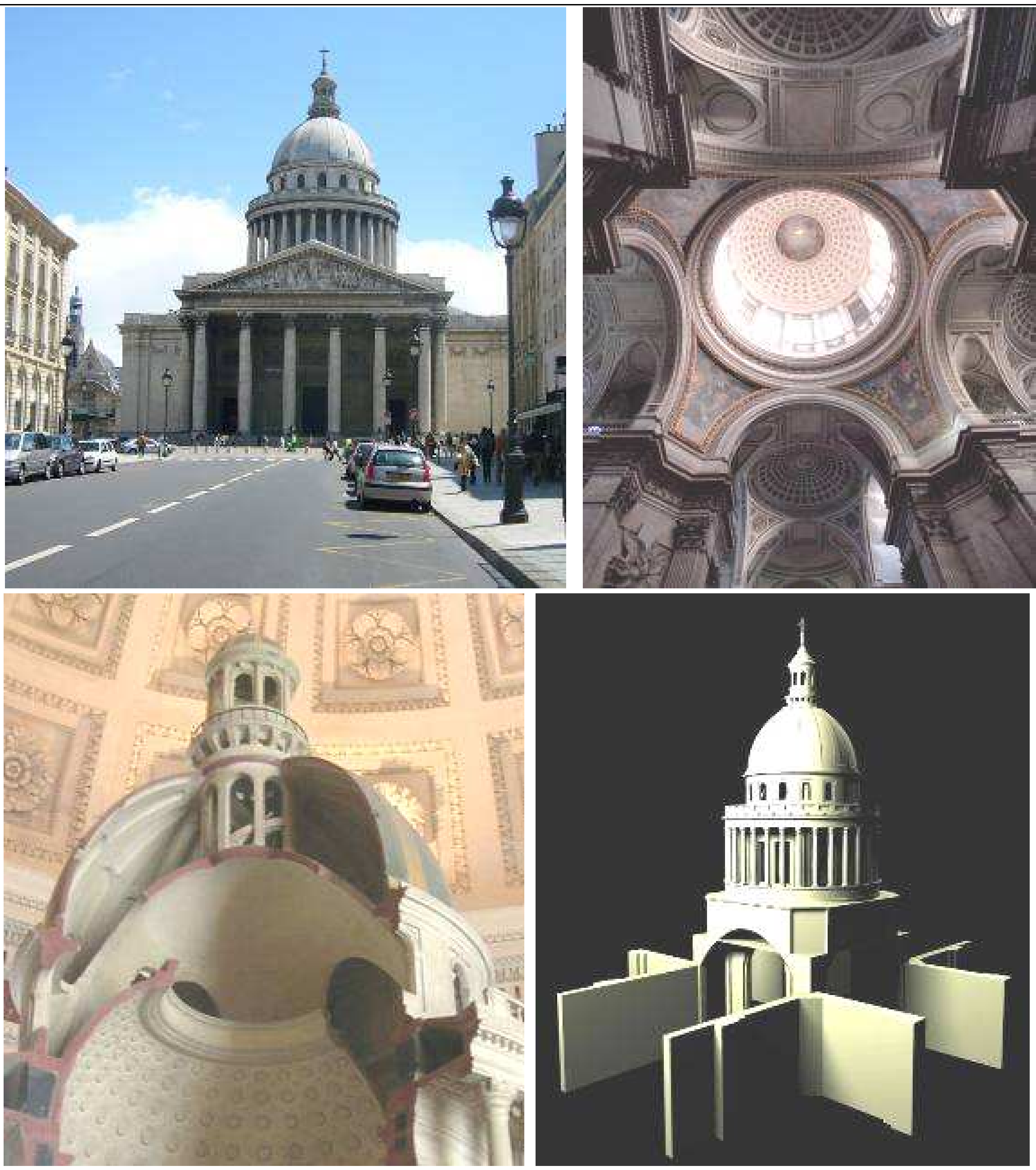

a) b) c) d)

Figure 1 – (a) The front of the Panthéon. (b) The inside view of the domes. (c) The model made by Rondelet shows the three superimposed domes and the lantern. (d) The numerical model shows the outer colonnade, resting over the great arches.

These unprecedented proportions triggered the first objections, still in the design phase, from the architects linked to the old academic building tradition (Patte 1770). Moreover, cracks appeared since the first building phases, particularly in the pillars sustaining the triple dome. The inevitable critics lead to a famous debate between designers and objectors, stimulating inspections and calculations which are still a milestone in building science history. Indeed, to assess the stability of the pillars, structural calculations and tests on materials were carried out to measure the ultimate strength, thus we can say that this is the first building that was "calculated" with a modern methodology.
In the last decades, stone pieces have started to fall from the vaulted ceilings. The French Ministry for Culture has therefore funded a study campaign on the stability of the monument, aimed at its consolidation[4].

4 The research group, coordinated by Prof. C. Blasi, was composed by professors of different disciplines from the University of Parma as well as experts in buildings humidity. Apart from the authors, the other participants to the

Within this campaign, accurate surveys, tests, numerical modelling have been carried out, but most of all the history of the monument has been studied, particularly focusing on the debate on the stability of domes and pillars which developed in the late XVIII century (Blasi 2005).

2. THE ROLE OF THE FRENCH PANTHEON IN BUILDING SCIENCE HISTORY

2.1. The “Art” of building and the “Science” for structural analysis

During the second half of XVIII century, architecture developed in France toward a joint appreciation of classical majestic shapes and Gothic slender and daring structures. At the same time, the industrial revolution introduced new materials, particularly iron, allowing new building technologies and stimulating the creativity of architects (Carobbi 2011).
Concerning the dimensioning of structures, the evolution from the “Art” of building, based on experience and classical rules, to the “Structural analysis”, able to foresee the mechanical behaviour of new materials, had in this period a rapid development. During the successive two centuries in the universities only the new science would have been studied.
The construction of the French Panthéon, beginning in 1756, became the object of a cultural debate and of scientific research within this scenery: slender shapes creating scandal among the academics of that time, widespread use of iron clamps and tie rods whose behaviour was not tested before. Only the structural intuition, innovative calculations and obstinacy of great architects and engineers (Soufflot, Gauthey and Rondelet) allowed these great results to be achieved.
The famous harsh debate which was ignited by the Panthéon disorders raised scientific issues and stimulated innovative calculations which were the base of the researches made in the following years on the stress-strain relation, on the mortar shrinkage, on plastic phenomena, on bending moments and on hyper-static problems (Heyman 1985).
The contribution of French architects, engineers and mathematicians, between the end of XVIII and the beginning of XIX century was fundamental in the development of the new “Building Sciences”.
Of course, the knowledge on the domes stability has developed greatly since then, but the debate about the stability of the Panthéon's domes is still very topical, as it testifies the deep empirical knowledge and the intuition that experts had at that time on the behaviour of masonry structures, and that were thenceforth partially forgotten.

2.2 The first tests on materials

The use of iron and the consequent abandonment of the intuitive classical shapes and proportions meant for the designers to abandon the well-known road of equilibrium and to understand that for each static problem, infinite solutions are possible and that the role of materials strength becomes fundamental.
For the first time tests on materials strength and deformability were carried out, in order to calculate the permissible stresses and evaluate the safety factors, demonstrating the theoretical correctness of the innovative structural dimensioning.
Particularly, the calculation of the stress state in the four great pillars sustaining the domes can be considered one among the first examples of modern structural analysis.
During their construction, Gauthey (no wonder he is the uncle of Navier) invented and built a machine (Figure 2) to test in compression and in indirect traction (three-point bending) many stone essays, measuring not only the strengths but also the strains of stones and mortars, and observing the shape of the cracks, looking for the effects of the stress direction (Gauthey 1798). Also Soufflot and Rondelet built similar machines and measured the stones strengths.

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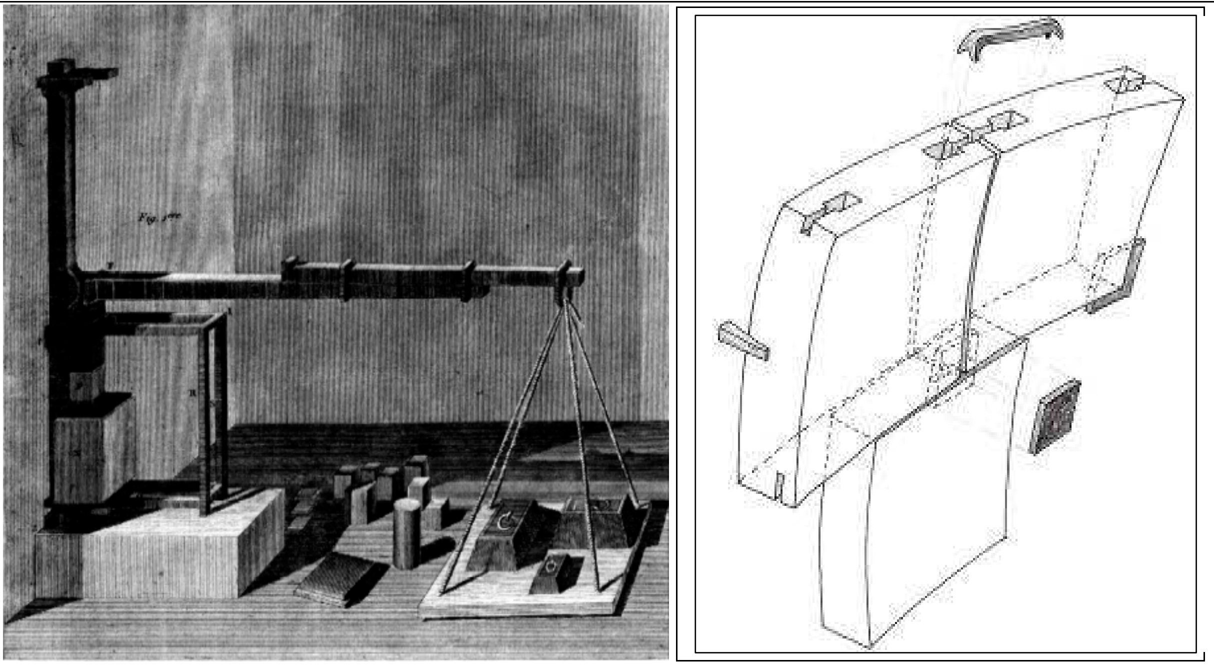
study were Ivo Iori and Daniele Ferretti (structures modeling), Gianni Royer (stone fractures modeling), Paolo Giandebiaggi and Andrea Zerbi (surveys), Alessandro Mangia and Giampiero Venturelli (materials analysis), Margherita Ferrero (geotechnics), Paolo Bresci and Leopoldo D’Inzeo (internal climatic conditions), whit the local support of Sandrine Voyer. The group collaborated with the “architects en chief of the monument” Hervé Baptiste et Daniel Lefèvre



a) b)

Figure 2 – (a) The testing machine made by Soufflot in 1775, based on Gauthey's previous invention, to prove the validity of the pillars dimensioning (Rondelet 1827-1832). (b) The "pierre armée" technique applied to the domes construction.

2.3 Do domes thrust?

Although the correct dimensioning of the pillars (under a centred load), had been demonstrated, cracks appeared during the construction. The debate then moved on to a second issue: the eccentricity of the pillars load, and therefore on the behaviour of the domes and on the possible horizontal thrust the pillars received from the domes.

Gauthey, Director of the "Ecole des ponts et chausses", knew, like all the experts, that hemispherical masonry domes do thrust on their supports, and suggested the insertion of buttresses (Figure 3 a) to limit the bending moments on the pillars, which he calculated roughly, only based on intuitive equilibrium principles (Gauthey 1798).

Rondelet, instead, wrote in 1797 that "spherical domes have no thrust" because masonry should first "rip", and this is not possible in a well reinforced and tensile resistant dome (Rondelet 1797): a revolutionary assertion! Besides, a manuscript credited to the young Rondelet objected "the validity and applicability of the formulas for the vaults equilibrium not considering the materials properties and their application procedures" (Guillerme 1989). Clearly Rondelet looked at the dome in reinforced stone masonry as a membrane, made of a tensile resistant material, largely anticipating the membrane theories.

Rondelet was right for domes with encircling tie rods. Nevertheless it would have taken decades before this would be demonstrated analytically. However, Napoleon entrusted Rondelet for the consolidation of the pillars (Figure 3 c) increasing their cross sections in 1806.

The uniform crack pattern on the pillars (surveyed by Rondelet before the consolidation works) demonstrated that the stresses were similar on all the pillars faces and thus the thrust of the domes was thus negligible.

Rondelet was right in the debate about the domes stability, and Soufflot (who died in 1780) had a correct intuition when he designed these shapes: the thrusts of the domes are taken up by the iron clamps and the encircling tie rods (Figure 4) and these guarantee the stability of this audacious monument.

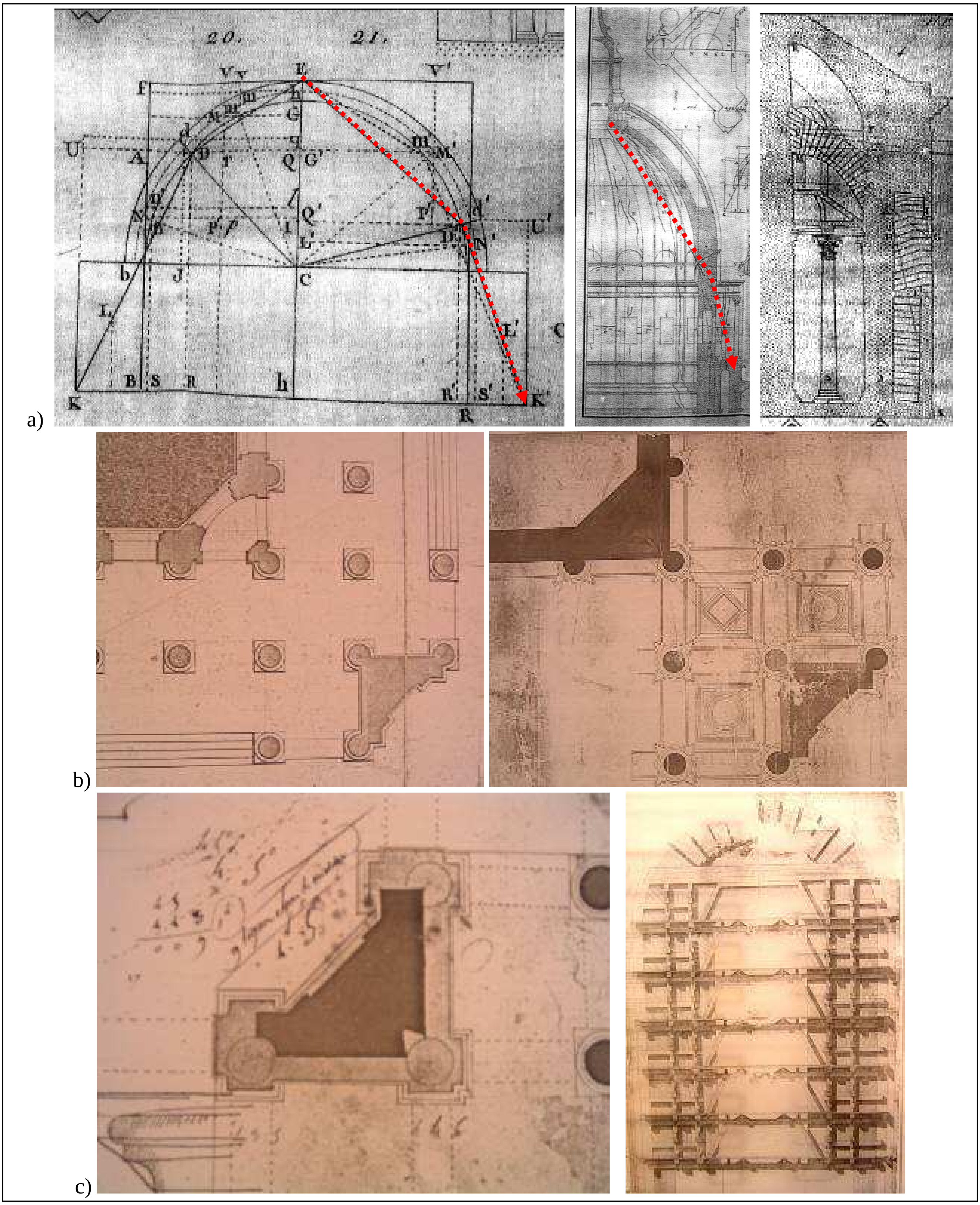


Figure 3 – The different proposals for the strengthening interventions made by Gauthey (a) (buttresses) and Rondelet (b) (windows plugging) and (c) (pillars enlargement) reflected their different opinions on the issue of domes thrust.

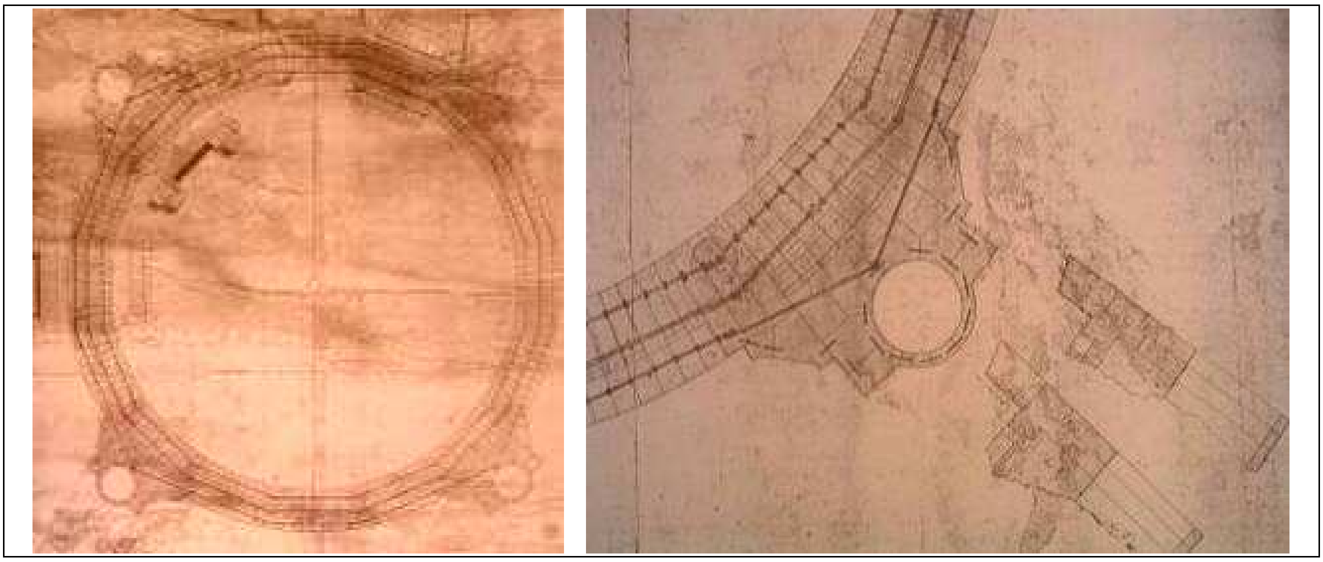


Figure 4 – The encircling tie rods in a dome horizontal section (left) and other particulars of iron elements inside the masonry (right) (Rondelet 1797).

3. STRUCTURAL PROBLEMS, FROM THE BEGINNING TO PRESENT TIMES

To understand the disorders which developed in time and caused the fall of stone pieces from the vaulted ceilings, a full comprehension is needed of the complex structural organization and particularly of the support system of the domes, partially relying directly on the pillars and partially on four great arches, over 30 m wide (Figure 1d).

In the Panthéon, three main structural systems were identified (Figure 5): the three domes and the underlying pillars, the four great arches which bear the tambour and rest over the outer walls, the inner isolated columns and the vaulted ceilings. These three systems have independent structural behaviours, but of course they are not completely separated: the links between them are the elements that show most problems.

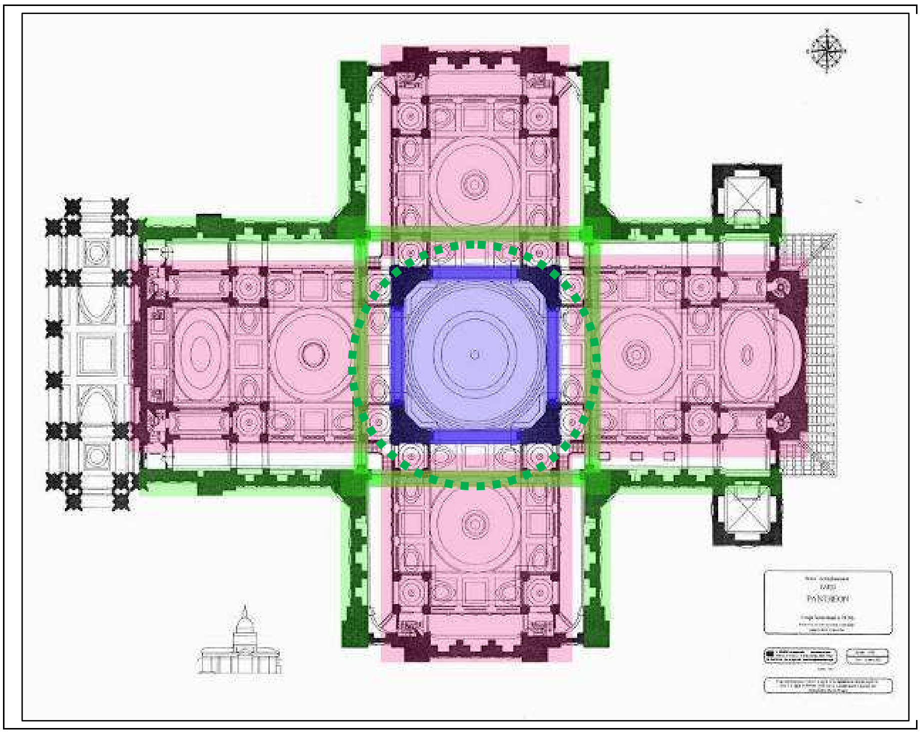


Figure 5 – The three structural systems of the French Panthéon: in blue the three domes and their pillars, in green the great arches and the outer walls, in pink the columns and the vaulted ceilings (Blasi 2005).

3.1. The crack pattern in the vaulted ceilings

Despite Rondelet's interventions, new widespread cracks appeared in the following centuries, and particularly in the last decades of XX century, when several stone fragments fell from the vaulted ceilings, causing the partial closure to the public for safety reasons.
The cracks mainly occurred in correspondence of the iron clamps and these were always oxidized: the cause of the rapid increase of damage was then initially ascribed to the decay of roofs and fixtures and to the consequent seepage of water, which oxidizing the metallic elements, caused their volume increase and the expulsion of the adjoining stone pieces.
The following waterproofing interventions did indeed reduce the detachments, but didn't stop them completely. The cause of the cracks could not, in facts, be ascribed only to humidity problems.

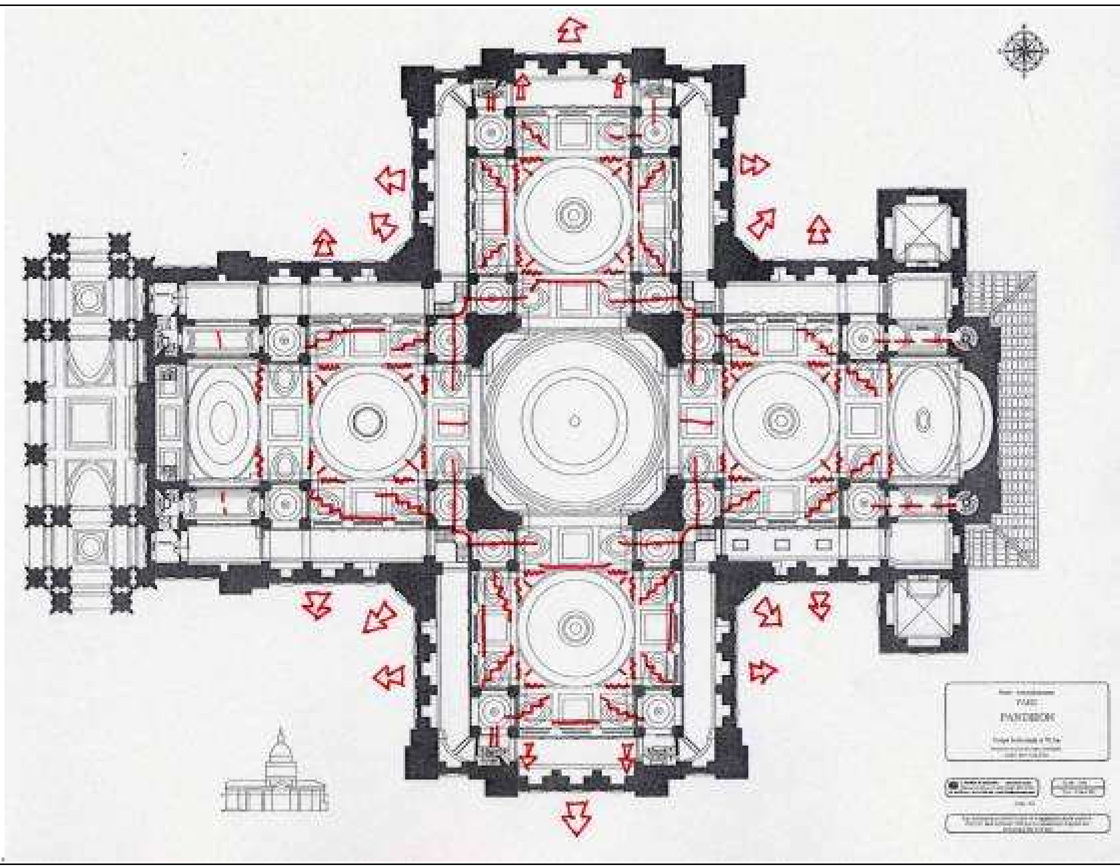


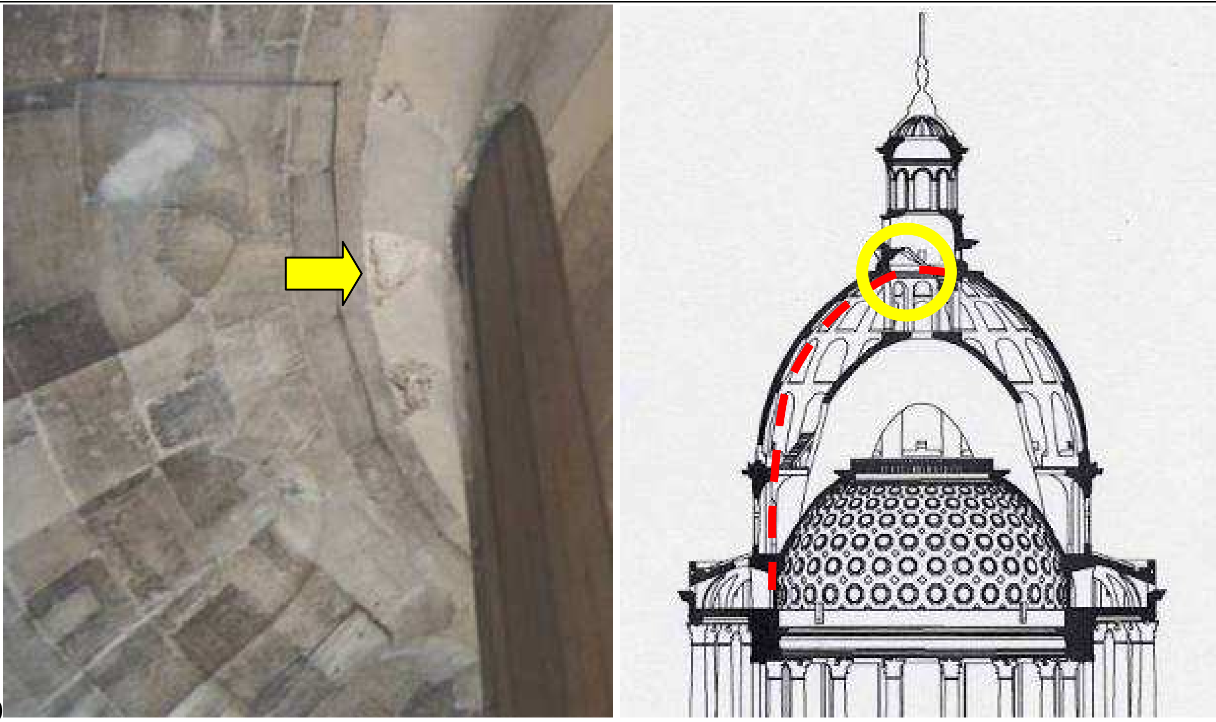
Figure 6 – The crack pattern survey (Blasi 2005).

To fully understand the origins of the deterioration phenomena, a survey of cracks was made (Figure 6), together with a detailed survey of the walls deformations in time. This showed that the crack pattern was very symmetrical, including areas not reached by water. The substantial symmetry of the crack pattern could not be ascribed only to accidental causes like the water seepage. Looking at the crack pattern survey with an eye to the structural behaviour of the monuments, it can be noticed that cracks are mostly located in the ceiling areas in which the masonry is subjected to traction, due to the absence of continuous tie rods. The tensile stresses are taken up by the iron clamps, but these seem to apply on the adjoining stones local stress concentrations. The origin of the cracks is therefore to be found in symmetrical structural movements of the monument, which are integral to the structure itself, combined with the fragility of the clamp-stone connection.

3.2. The domes disorders

The three domes are at present in a fair condition: no worrying disorders can be observed; only minor cracks can be seen both in the inner and in the outer domes, connected to the tensile stresses in their lower parts, naturally concentrated in the thinner parts. Also the cracks in the lower tambour seem to be caused by normal

settlements: an instrumental monitoring did not show any significant movement. Rondelet had already noted that the domes were all well reinforced and that the static thrusts were taken up by the numerous encircling tie rods. Of course, Rondelet's encircling tie rods were not in high strength steel and were not adequately pre-stressed, compressing the masonry, thus they could not hinder the formation of small cracks in the masonry, given the natural elongation of the tie rods. Also the thermal variations must have had an influence on the effectiveness of the tie rods and on the behaviour of the domes.
The only crack pattern which deserves more attention in the domes is the one that can be observed in the stones between the lantern and the external dome, as pieces of stones continue to fall from there (Figure 7a).
As Rondelet himself wrote (Rondelet 1797), the catenary shape of the intermediate dome, the rounded connection between this dome and the lantern and the stiffness ratio between the intermediate and external domes indicate that the designer clearly entrusted the load of the lantern to the intermediate dome. The cracks that can be seen at the contact between the external dome and the lantern suggested to some that part of the lantern load could be transmitted by shear stresses to the external dome. Indeed, the large difference in vertical stiffness between the two domes suggests that the external one can only take up a minimal part of the lantern load.
Looking for other possible causes, new temperature measures have been taken, showing that in the gap between lead and stone, temperatures over 60 °C can be reached. It was then clear that these large thermal variations, over both daily and seasonal cycles, can bring to large differential movements between the outer dome and the intermediate one, that is subject to much lower temperature changes than the outer dome, but it is strictly linked to it (Figure 7b).



a) b)

Figure 7 – The cracks at the intersection of the intermediate and outer domes (a) can be explained by the thermal deformations of the domes (b): the external dome undergoes much higher thermal variations than the intermediate one and the contact area is therefore highly stressed.

4. STRUCTURAL IDENTIFICATION AND PROBLEM SOLVING

4.1. Modern and historical surveys

The control measures and surveys on the Panthéon have began since its construction, in order to try to understand the causes of the cracks which appeared so prematurely.
The first surveys were made by Rondelet, starting in 1780 (before the domes were completed), on the four great pillars and on their disorders (Rondelet 1797).

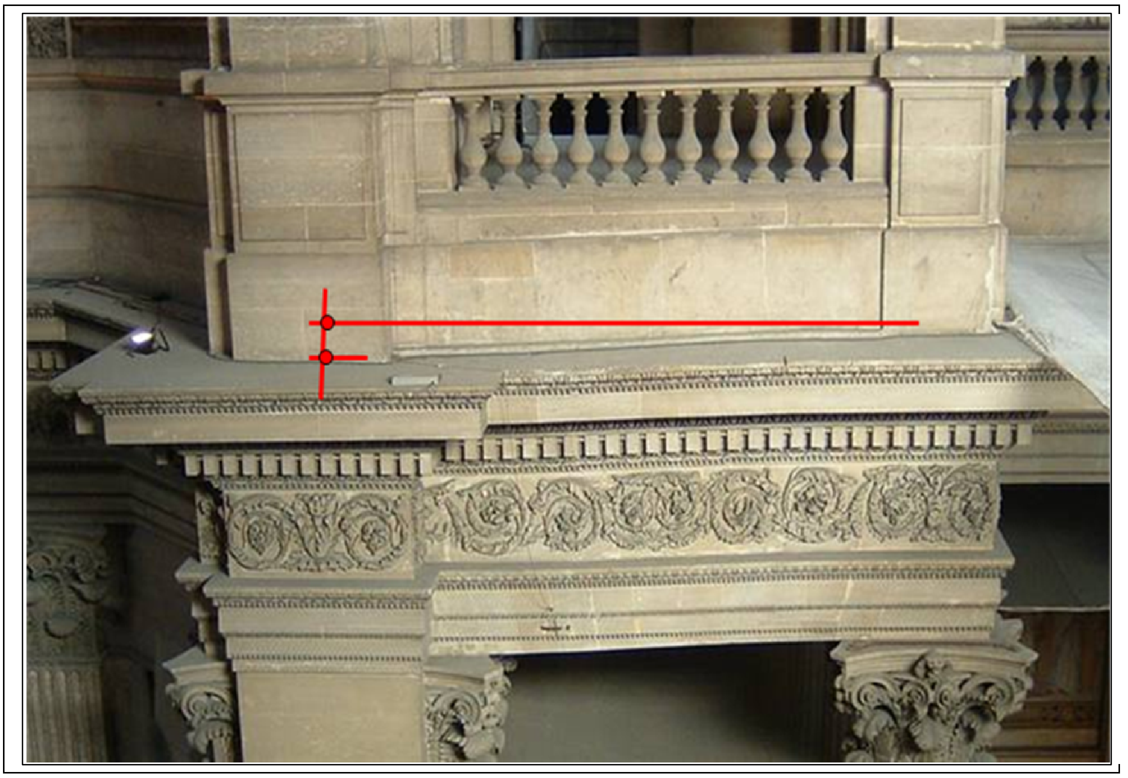


Figure 8 – The incredible settlement of one of the pillars sustaining the domes was, at Rondelet's times, 122 mm; now it is 210 mm, despite Rondelet's strengthening works. These data prove the two basic phenomena of the masonry: the very low stiffness and the plastic strains also in very long times.

New measures were taken by Rondelet and Gauthey up to 1798; Gauthey also installed some pieces of plaster and of paper on the pillars cracks in order to control their evolution in time (Gauthey 1798).
Gaspard Riche de Prony has tried since 1798 to measure the global droop of the dome, with a system of ropes, but he had great difficulties due to the temperature effects.
In 1972 plaster crack monitors were placed on the ceilings to control their openings. In the 1980s a monitoring system (not automatic) was installed and (after a long period of inactivity) it was checked again to see the increments in the last 25 years (Figure 9). No instruments could be placed in the vaulted ceilings, given the difficulties in collecting the data.
All the informations that can be collected from these surveys and ancient and modern crack gauges are precious to understand the evolution of the disorders in the monument and to evaluate the present safety conditions. Most of the monitored cracks (IGN 1987) now seem subjected only to cyclic, seasonal variations (which, however, can also be dangerous). Only the cracks in the round galleries below the domes still showed some increment in the last 25 years.

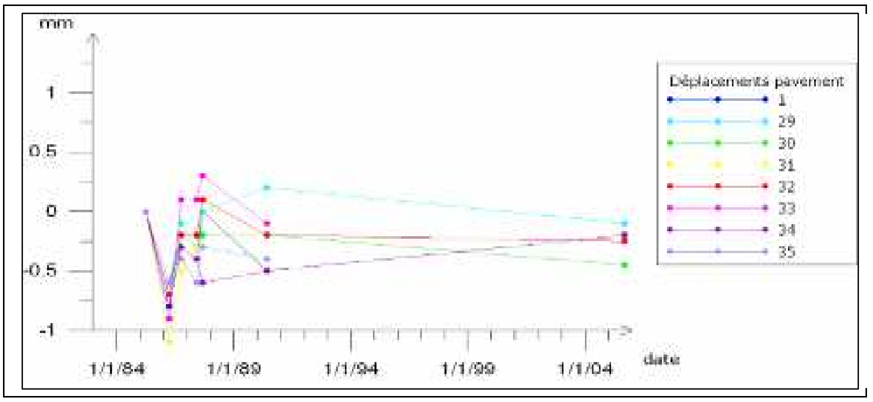


Figure 9 – The monitoring data from the modern crack gauges (Blasi 2005).

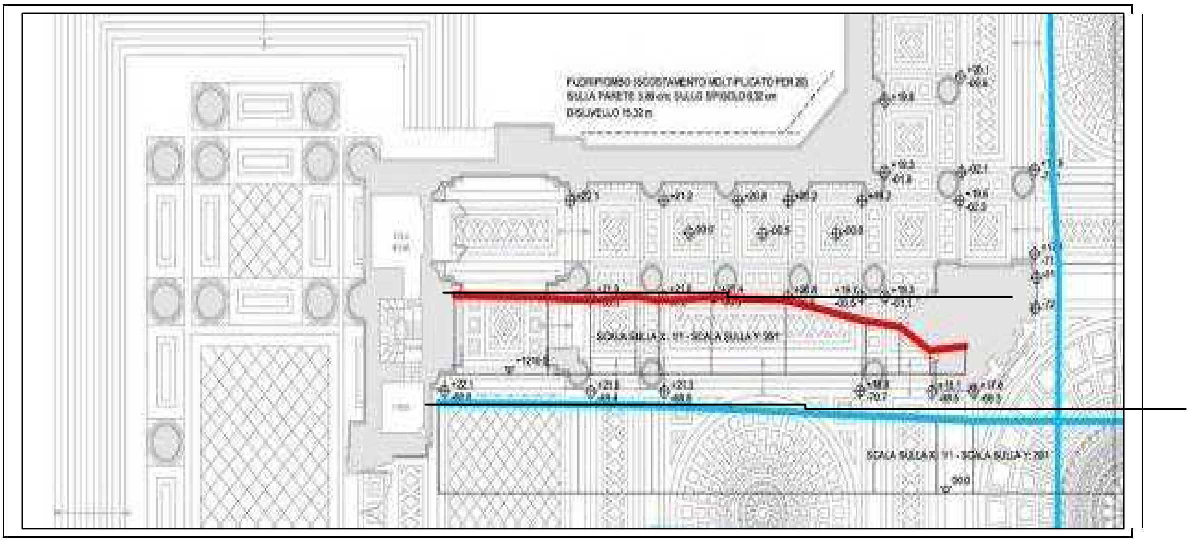


Figure 10 – The settlements of the bases (blue) and of the top (red) of the columns (Blasi 2005).

Moreover, the level surveys carried out in the XVIII century were compared with new specific surveys on the global movements of the structure (Figure 10), given the hypothesis that the geometries were originally precise (Rondelet wrote that all arches were built with a camber to compensate the load deflection). This was a unique opportunity to control the settlements and deformations after 200 years, in a methodological continuity with our predecessors.
The main results of these surveys and comparisons are the measure of a symmetrical 4 cm settlement below the central part of the monument, where the weight of the domes rests on the ground through the great pillars. The soil settlements can be therefore considered normal for the weight of the building and the different loads on the various parts, but Rondelet was right when he affirmed that the contribution of these settlements to the structural damages was negligible.
Nevertheless, the surveys (both ancient and modern) show significant differential settlements between the top of the pillars and the other parts of the building. If the soil and the foundations have been cleared of the accusations, the origin of these settlements must be searched for in the pillars themselves and in the deformability of the masonry that composes them.

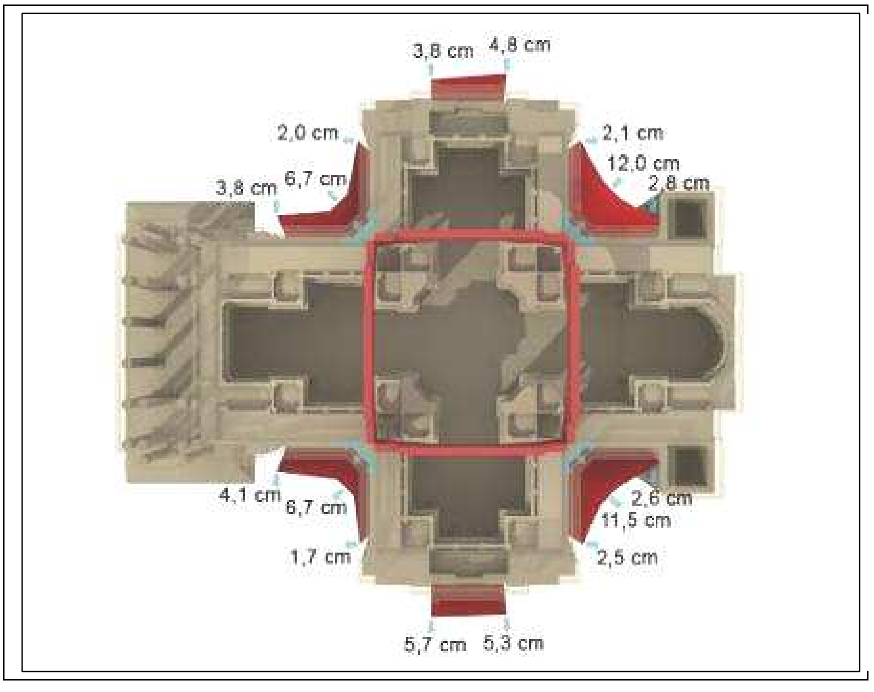


Figure 11 – The tilts measured on the outer walls (Blasi 2005).

Lastly, the survey of the outer walls (Figure 11) has shown interesting movements. The survey shows totally symmetrical outwards tilts of 6 to 12 cm, as represented in figure 10. These movements, which easily justify the cracks in the walls and the adjoining ceilings, are connected to the thrust of the great arches, which partially sustain the outer dome, and to the deformability of the outer walls in the own plan.

4.2. The problem of deformability

The described surveys have shown that the Panthéon's masonry, even if apparently very well made, has a very low stiffness. In particular, the pillars "shrunk" of 21 cm over a height of 14 m: this huge deformation can be theoretically explained only considering a secant elastic modulus of 200 MPa, about 20 times less than the one that can be found with short time tests on stones and mortars. This value becomes plausible if we read Rondelet's description of his inspections on the pillars (Rondelet 1827-1832): the masonry appears externally perfect, with mortar beds only a few millimetres thick, while inside the stone blocks are less carefully squared and the mortar beds reach a few centimetres (Figure 12). Only considering this fact and reducing as a consequence the elastic modulus, the movements in the structure could be reproduced in the numerical models. We cannot know whether the increase in settlement from 122 mm (measured by Rondelet in 1797) and the present 210 mm happened between 1797 and 1806 (when Rondelet strengthened the pillars) or up to now. Given the cracks in the architraves adjoining the pillars, the latter hypothesis looks like the most probable, but this induces some important reflections on the fact that masonry can continue to deform for centuries, on the basic role of plastic strains on the mechanical behaviour of masonries and on the consequent difficulties in numerical modelling of historical buildings.

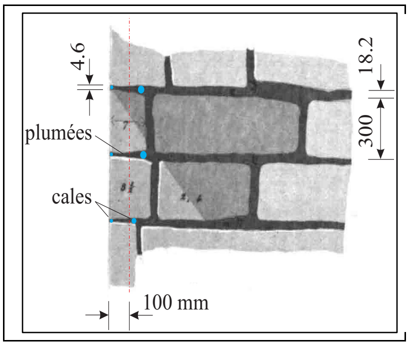


Figure 12 – Rondelet's inspections in the pillars masonry show the large difference in the mortar bed thickness between the outer and inner masonries (Rondelet 1827-1832).

4.3. Finite element analysis of temperature effects on the domes

For a better understanding of the cracks described particularly in the connection between the outer dome, the intermediate dome and the lantern, a finite element analysis has been carried out. This model of course cannot be considered exhaustive, as the knowledge of the materials characteristics, of the damage pattern, of the load history is not complete, but it can give clear indications on the static behaviour of the domes and their contribution to the sustain of the lantern.

A simplified 3D model was made of the two domes, the lantern and the upper tambour, with the ABAQUS code. The mechanical behaviour of the masonry was refined step by step: first a linear elastic model to roughly check the values, then a non linear model to identify the cracked areas and compare them with the surveyed ones, finally a non linear analysis with the insertion of unilateral joints along the cracks.

Also the encircling tie rods indicated by Rondelet in his drawings in and around the domes were modelled. The braking of the most stressed tie rod has been simulated to evaluate the increase in stresses in the other ties and assess the possible triggering of a chain reaction. Also the possibility of breaking of all the encircling tie rods has been simulated.

The supports at the base hinder only the vertical movements (as major soil settlements have been excluded by previous inspections), while rotations and radial displacements are allowed.

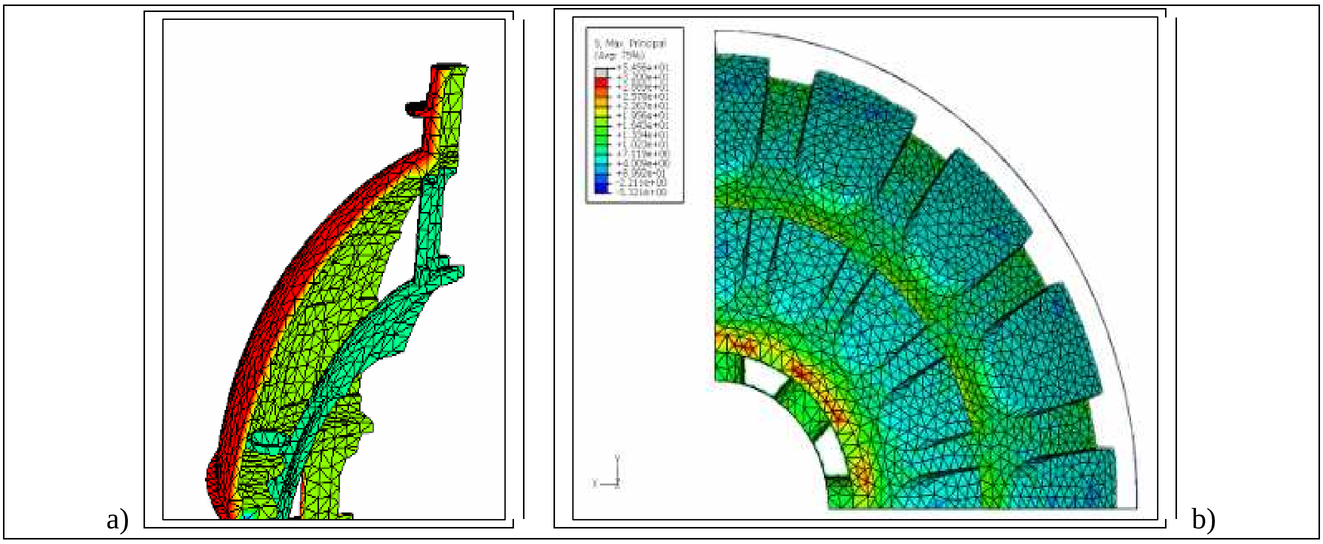


Figure 13 – The thermal, non linear finite element analysis of the two outer domes: the model with input temperature (a) and the tensile stresses in the outer dome seen from below (b), which show concentrations in the connection with the lantern.

First only the self weight was applied and the results indicated very low stresses in the ring around the lantern. The first hypothesis of the shear stresses between the two domes being at the origin of the cracks in this area seems therefore unfounded.
A thermal analysis on the non linear finite element model has then be carried out to check the validity of the second hypothesis, which linked the cracks to the thermal variations. The analysis highlighted large tensile stresses in the ring that connects the two domes where the lantern starts, when the expected thermal changes occur (Figure 13). This area is therefore subjected to cyclic tensional stresses, and it corresponds to the areas where regularly new cracks can be seen in the stones, demonstrating the importance of environmental thermal variations on these large monumental buildings.

4.4. The proposed minimum intervention

In restoration, one of the basic principles is the “minimum intervention”. In structural restoration this principle means to supply the least help which is needed for the physiological functioning of the structures. To do this it is mandatory to understand first the original structural behaviour.
In brief, as Rondelet has already solved the problem of the cracks in the pillars masonry, the main problem of the Panthéon consists nowadays in the high deformability of the masonry under long time loads and in the presence of iron clamps and tie rods, which, on the contrary, make the masonry fragile; iron clamps, being more rigid than the adjoining masonries, create stress concentrations and thus cracks. The fact that the reinforced stone masonry technique, which developed in Paris at the end of the XVIII century, was soon abandoned (Carobbi 2011), is a symptom that architects understood that the compatibility problems between stone and iron weren't solved yet.
In particular, in the Panthéon, the cracks showed up in those building elements and in those areas where the effects of the deformations were higher, as higher were the loads applied: in the pillars, for the high compression stresses, in the dome, for the radial thrusts, and in the vaulted ceilings, deformed by the thrusts of the great arches. For the pillars, Rondelet has to a large extent solved the problem increasing their cross section and lowering their deformability. For the radial thrusts of the domes, the numerous encircling tie rods have allowed only small cracks to create, until the tie rods started working. The strengthening project which is under approval now consists in the insertion of three more external pre-stressed encircling tie rods, made with high strength materials, invisible from outside and easily removable (Figure 14). The new tie rods will constitute a safety element for the future, in case the original tie rods would oxidise or brake. The compression induced by the new tie rods on the masonry will also reduce the thermal strains.
The only important structural elements which have no active elements to take up their thrusts are the great arches. These thrusts are the cause of the deformations of the outer walls and of most of the cracks in the vaulted ceilings (Figure 15). The challenge in years to come will be to find an active system of tie rods able to take up the thrust of the great arches and compatible with the monument (Figure 16).

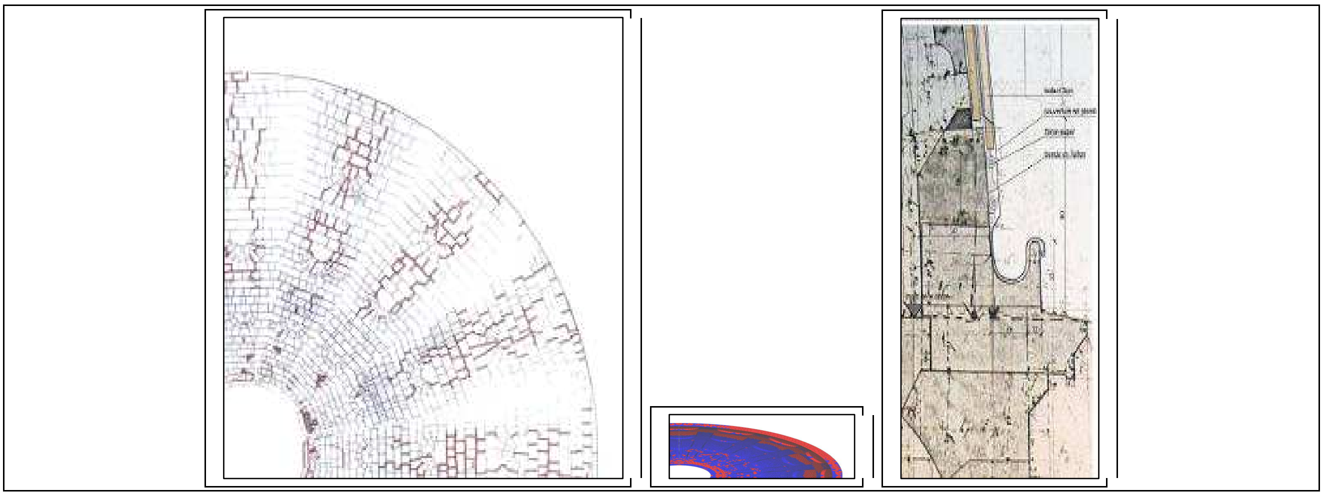


Figure 14 – The small cracks in the outer dome, the numerical model (in red the cracked zones), the new prestressed ties at the base of the outer dome.

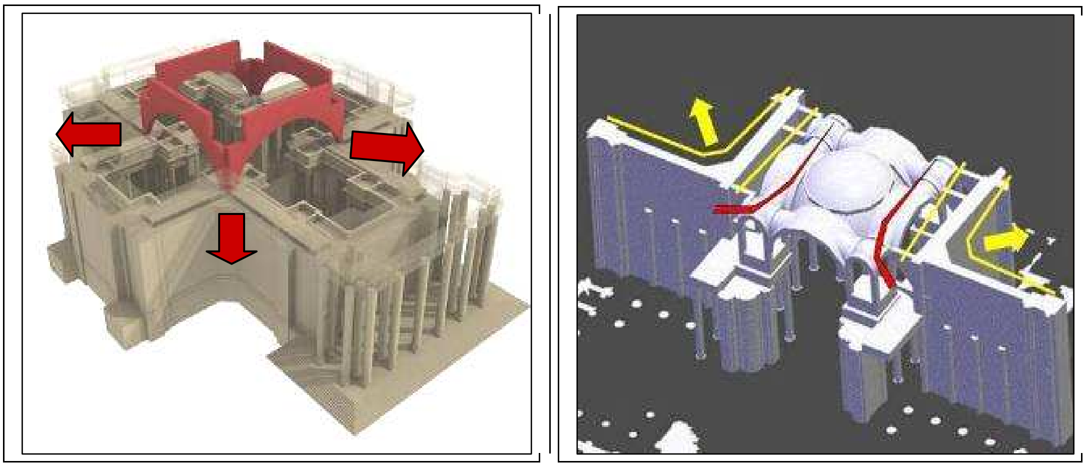


Figure 15 – The thrusts of the four great arches and the deformations in the outer walls and in the adjoining structures.

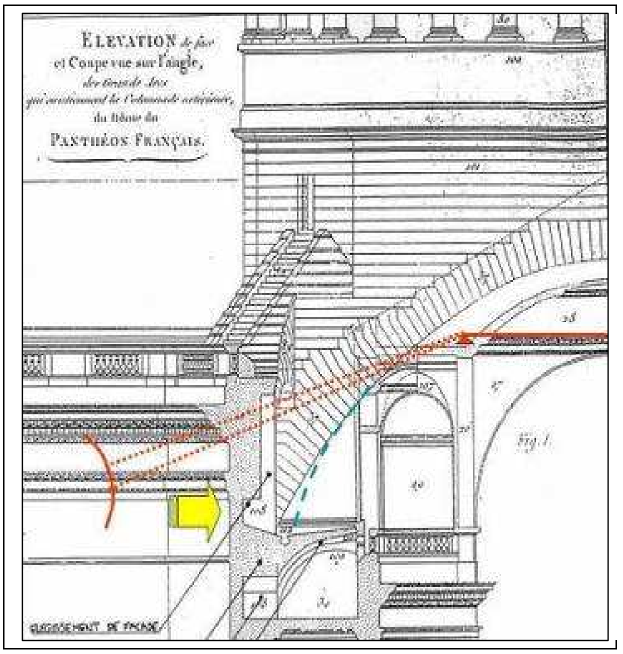


Figure 16 – The possible positioning of tie rods to take up the thrusts of the great arches.

ACKNOWLEDGMENTS

This research has been funded by the French Ministry for Culture and Communication.

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