Corbelled domes structural behavior

Luisa Rovero[1], Ugo Tonietti[2]

Abstract In the Mediterranean area, buildings with roofs arranged as corbelled domes comprise a widespread and greatly valuable heritage that deserves protection and enhancement. Unfortunately, structural behavior of corbelled domes has been investigated only to a limited extent. In particular, in the most known approach, the so-called corbelling theory, analysis is formulated by imposing the balance of the overturning of infinitesimal dome meridian wedges considering only a vertical forces transmission. The application of the corbelling theory in two real case studies shows that the theory is not able to capture the structural behavior of the corbelled dome. The paper highlights that the key factor of the structural behavior is the horizontal action along the parallels of the dome, produced by the engagement and friction between the blocks, or from the cohesion if there is mortar. A modified corbelling theory is proposed to take into account, in structural analysis, the actions along the parallels.

Keywords corbelled dome, structural behavior, cultural heritage safeguard

1. INTRODUZIONE

1.1 A spread heritage

Buildings covered by corbelled domes, consisted of purely horizontal layers slightly cantilevered toward the center until meeting at the top, are still widespread in many Mediterranean countries. Despite its spread, it is a kind of dome (also named "false dome") not really known, that met the interest of the scientific community only in rare cases. Many monumental buildings have been the subject of archaeological studies that highlighted their historical and social value (Wace 1921; Mylonas 1966; Pelon 1976; Besenval 1984); in this case the construction techniques, the link with the territory and the available technology, the relationship between artifact and social issues have been enough investigated (Palyvou 2009). On the contrary, for a long time, the vernacular buildings were considered as insignificant because classified as a simple expression of rural life.
Every building is characterized by its regional membership and, consequently, by the building material. It is just the material that separates two great realities in the Mediterranean area: in the north the dry stone constructions and in the south the earth constructions. Where the stone material prevails, the dislocation in the territory is strongly linked to the geomorphologic characteristics and to the presence of outcropping formations. As regards the earth constructions, the availability of clayey soils

[1] Luisa Rovero, Dipartimento di Costruzioni e Restauro, Università di Firenze, Italy, luisa.rovero@unifi.it
[2] Ugo Tonietti, Dipartimento di Costruzioni e Restauro, Università di Firenze, Italy, u.tonietti @unifi.it

determines their greatest development in some areas than in others. In Europe stones artifacts prevail (Paglini 2008).
Two relevant typologies can be identified: the hypogean constructions (with mound) and the buildings with a domed roof without any mound. In the first case Mycenaean tholoi (Grece) are, undoubtedly, a very high example as regards the constructive technique (Palyvou 2009) (Fig.1). The same type of construction is found with similar formal characteristics, though less impressive, at Populonia in Italy (Etruria), where the tombs of the Necropolis of Granate and those of the Necropolis of San Cerbone (Barbi 2000), are the most important examples (Fig.2).

Figure 1- The Treasury of Atreus of Tomb of Agamemnon, Micene, Grece, 1250 BC.

Figure 2 - Tomb of Carri Necropolis of San Gerbone, Populonia, Italy, VI-VII century BC, after the restoration.

The building with a stone domed roof, without mound, are widespread in many parts of Europe (Juvanec 2001, Juvanec 2009). In this case the development of the technique is less easily traceable. In many countries such buildings are mostly related to the development of rural life, associated to the need of shelters for pastoral activity. Establishing their origins is therefore not immediate nor certain.
In addition to this vernacular heritage, monumental examples of great value are the ruins of corbelled domes that covered the rooms of the megalithic temples of Malta, which are dated around 3500 BC (Fig.3).

(A) (B)

Figure 3 - The ruins of (A) Mnajdra temple, Malta, 3500 BC and (B) Hagar Qim temple, Malta, 3500 BC.

In many European and Mediterranean countries (Malta, Italy, France, Spain, Germany, Slovenia, England, Croatia, Switzerland) – some examples of building with corbelled domes are characterized by significant architectural and visual impact, notable constructive solutions, so that they and can be considered as monuments. It is the case of the "Trulli" of Puglia, Italy, (Fig.4). Finally, a special case is represented by the Nuraghe (Fig.5), the great monumental buildings of Sardinia, Italy, that may be included as part of the cyclopean buildings (megalithic). On the Near Eastern Mediterranean the cultural tradition of corbelled domes is also very widespread in southern Turkey, Syria and Iran.

All these buildings covered by corbelled domes are surely the result of a process based on the self-building, but also an expression of a considerable constructive knowledge, handed down over time, linked to the land and so deeply rooted in the cultures of each country. In this way, they represent a building heritage to be studied in order to carry out plans of protection and enhancement.

Figire 4 - Trulli of Alberobello, Italy.

Figure 5 Nurage Santu Antine, Sassari, Italy, 1500 BC.

1.2 The literature

Literature concerning the structural behaviour of corbelled domes is not too full. Beginning from the 80's, some papers were focused on the analysis of Mycenaean *tholoi*, i.e. the monumental examples of corbelled domes like Atreo's tomb, characterized by the presence of a big earth bank as covering (mound). In addition to the studies of many archeologists (Pelon 1976), whose contributions were fundamental for the description of the building techniques typical of such artifacts, some contributions dealt with the problem of the structural behaviour (Cavanagh and Laxton 1981, Santillo and Santillo Frizell 1984, Como 2005, Como 2006).

In (Cavanagh and Laxton 1981) an in-depth structural analysis, applied to five Mycenaean *tholoi*, is carried out by means of the most shared theory, known as "corbelling theory"; in this theory, forces can be transferred only along the vertical direction, and consequently equilibrium is imposed, in a limit situation, on an infinitesimal meridian wedge of dome, by equating the stabilizing and overturning moments produced by the overhanging masses at any given level. In such a way, the possible cooperation in the horizontal rings among stones, because of interlocking and friction, is not considered. On the contrary, a "horizontal ring theory" was proposed in (Santillo and Santillo Frizell 1984), under the assumption that the stones are very tightly built and, consequently, they may be compressed in order to overcome the tendency to fall inwards. Such idea is recovered and developed

in detail by Como (2005, 2006), whose analysis demonstrated that only a membrane stress (and, consequently, the stress along parallels) can ensure equilibrium in the case study of Atreo's Treasure.
Literature offers only few studies dedicated to the structural behaviour of not hypogeum corbelled domes. Benvenuto and Corradi (1988) analyze corbelled arches, *tholoi*, and corbelled domes through the corbelling theory, based only on the force transfer along the vertical direction, and on the absence of ring collaboration. This theoretical model is a plane model, since the contribution to equilibrium, following the corbelling theory, is given only by forces contained in the mid-plane of the infinitesimal meridian wedge, without taking into consideration actions out of the plane. In other words, it is assumed that there are no static contributions related to the horizontal ring of the structure, as if it would repeat the structural model of a corbelled arch, without taking into account the 3D behaviour of the whole structure.
The corbelling theory is also applied by Fábrega (2003) for the investigation of Catalane domes, whose profiles are almost straight and, for this reason, approximately assimilable to cones. By assuming that the profile is straight, the author writes simple equations (that impose the overturning equilibrium), and so the relation among thickness, span, height, jut can be found.
The corbelling theory can be close to explain the equilibrium conditions for underground domes where the earth bank (mound) is useful for stability, but it is hardly able to explain the equilibrium of domes (without mound), which often exhibit also very thin profiles.
In the present study the structural behavior of the false dome (not underground) is investigated by two studies cases. As it is shown that the corbelling theory cannot be used to interpret the structural behavior of these domes, a modified corbelling theory is proposed in order to consider the static contributions related to the horizontal ring action. In addition to this, a numerical analysis with the Straus7 software was performed on the two case studies to investigate the stress pattern. Finally, a very simplified model of corbelled dome was built with bricks to highlight the role and engagement of the friction between the blocks.

Figure 6 - Exemples of vernacular architetture (from left and from 'high, row by row): Aleppo (Syria), Malta, Sardegna (Italy), Vaucluse (Francia), Sicilia (Italy), Sicilia (Italy), Puglia (Italy), Puglia (Italy), Puglia (Italy).

2. STRUCTURAL ANALYSIS

2.1. Two case studies

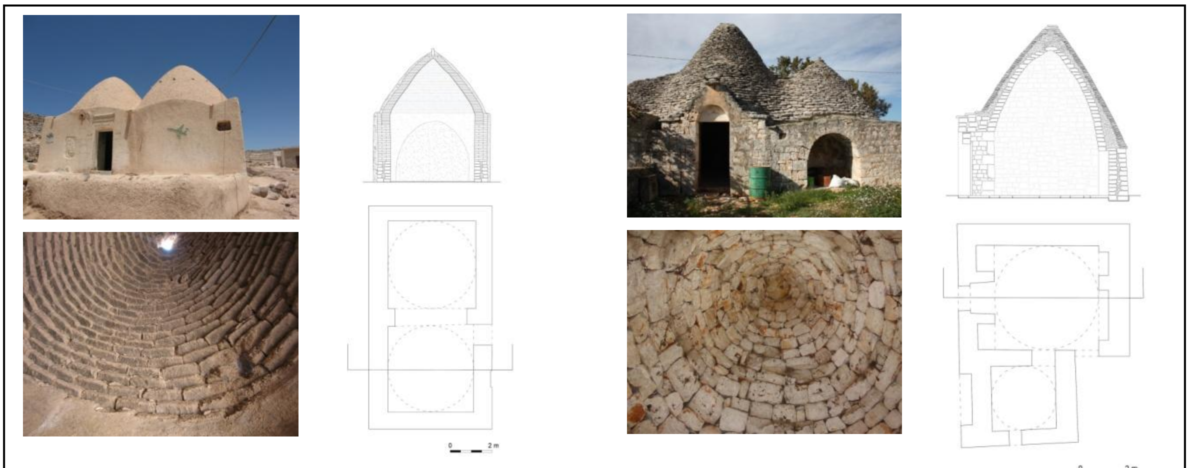
Two different case studies were considered: an adobe (earth brick) dome, typical coverage of dwellings in many villages (still inhabited) in the Aleppo's region (Syria) (Fig.7) and a dry stone dome, typical "trullo" building of the Valle d'Itria in Puglia (Italy) (Fig.8).

2.1.1. Earthen Syrian domes

The dome of a typical house in the village of Alrahib, placed near the salty lake of Jabboul, south-east of Aleppo, was studied (Paglini et al. 2008a, Paglini et al. 2008b). The house consists of two square cells, with a communicating arc in between, both covered with a dome, according to the typical pattern of the area (Fig.7A). This unit has been chosen as a case study, because it was representative of the type prevailing in the region, for its good state of preservation, and thanks to its isolated position (that allowed an easier survey process). The dome is built by superimposed progressively jutting rings, made with earthen bricks and earthen mortar. The thickness coincides with the largest dimension of a single brick (about 35-40 cm); thinner bricks are used (thick around 5-6 cm), if compared with the dimensions of the bricks used in the supporting walls. The use of thinner bricks enables surely a better control of the curvature by limiting the extension of the juts between the superimposed layers.
It is important to underline that the circular surface where bricks are laid is not perfectly horizontal, but endowed with continuity through bricks laying in a spiral progression. The slope increase is very small in the lower rings, and grows as the device progresses. This special building process, that is indeed remarkable in terms of geometry, does not alter significantly the hypotheses regarding the structural behaviour in the context of the so-called corbelled domes, even if it agrees with the idea of an apparatus-system, characterized by a remarkable cohesion.

2.1.2. Dry stone dome of "Trullo" building

A building covered by corbelled domes, well known as "Trullo", located in Castellana (Puglia, Italy) was analysed. This dome is perfectly representative of the typical roofing of the houses in the Itria valley (Fig.7B). Placed on a squared masonry room, whose walls are built by irregular stones, the dome is made up by stones too. The construction technique is easy: a single circular layer of stones complete each ring without any mortar; then the upper ring is placed on the lower jutting some centimetres (the corbelling can vary from 5 to 15 cm). The average dimension of the stones is about 30 x 10 x 15 cm, so the thickness of the dome structure is close to 30-35 cm at any level. Each ashlars is quite irregular but it is always slantwise cut in intrados face in order to show a rather continue surface of the inner side of the dome. A special care is dedicated to achieve the continuity of the ring brick-work, using little pieces of stones in order to fill any gap between blocks. In the extrados of the dome sloping flaked stones (30x10x3 cm) are subsequently put with the aim to protect the internal structure.



(A) (B)

Figure 7 - Cases studies: (A) Earthen Syrian domes; (B) Dry stone dome of "Trullo" building.

2.2. The corbelling analisys

As previously mentioned, the analysis follows the theoretical outline proposed by Benvenuto and Corradi (1988), based on the "corbelling theory". This model is based on the following assumptions: a) rigid and frictionless blocks; b) a building technique, that allows to consider any horizontal layer as a single block; c) no mortar (therefore an infinitesimal distance between adjacent layers). Following these assumptions, the authors determine the intrados and the extrados equations, in a "limit equilibrium" condition, by writing a complex set of integral equations (whose manual solution is possible only in special cases).

With reference to Fig.8A, for any M point belonging to the intrados curve, two regions are defined: MNP, that determines the stabilizing moment (M_s), and MPOA, that generates the overturning moment (M_r); the two contributions are calculated with respect to the rotation axis through point M.

The balance equation $Ms = Mr$ consists of the following terms:

$$Mr = \gamma \int_0^x [y(t) - Y(t)](x - t)t \; dt$$

$$Ms = \gamma \left(\frac{1}{6} [y(t) - Y(t)] \, h^2 \, x\right)$$

where γ is the specific weight, and y(x) and Y(x) represent the position of the points on the intrados and on the extrados (the latter being defined by the secant curve passing trough points P and N points, in order to simplify the expression of Ms) respectively.

Through a suited implementation in Mathematica 6, the integral equation M_s=M_r was solved, thanks to the transformation into a differential equation in the unknown function

$$z(x) = [y(x) - Y(x)]$$

that represents the difference between the positions of the intrados and the extrados. By means of the following simplification, that allows to establish the relation between intrados and extrados functions, and dome thickness (h), whose value is constant and equal to the brick length

$$\frac{[y(x) - Y(x)]}{Y'(x)} = h$$

the Y(x) function can be determined, and consequently y(x) function too, since $Y(x + h) = y(x)$.

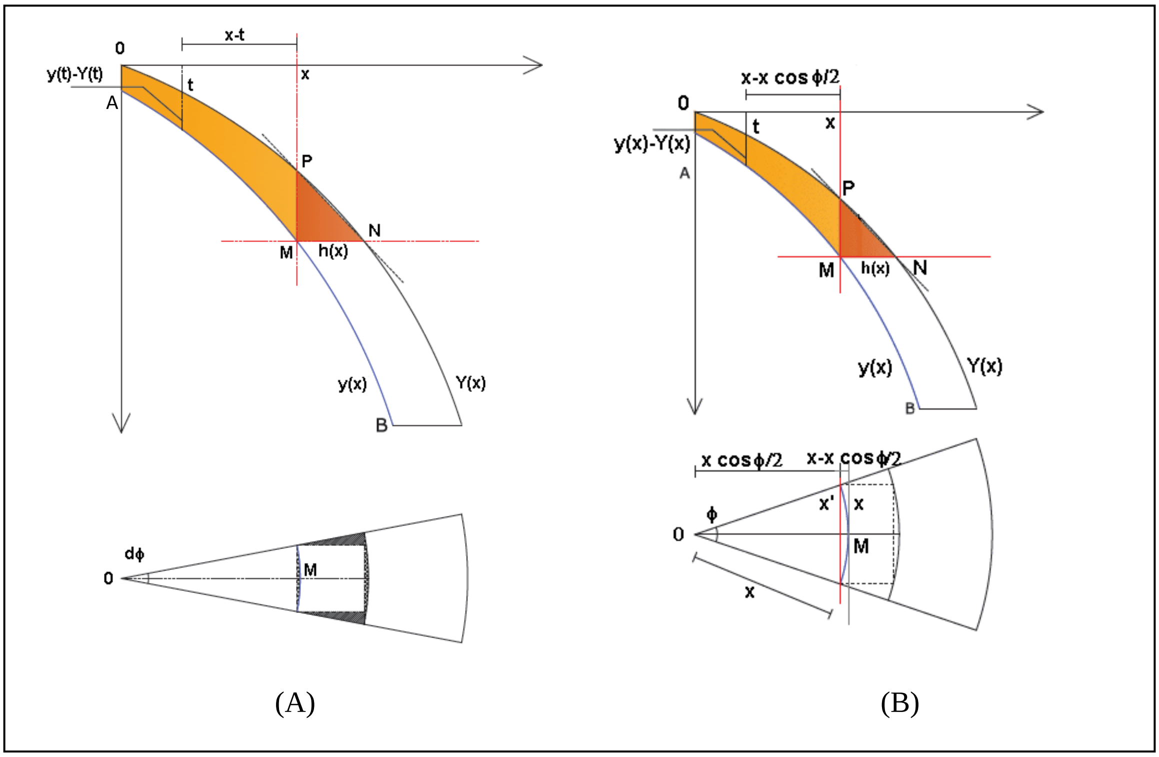


Figure 8 - Infinitesimal meridian wedge of a dome (A) and finite meridian wedge of a dome (B) to evaluate the equilibrium by the corbelling theory.

Fig.9 shows the two limit curves, representing intrados and extrados, obtained by the previous formulation, compared with the real ones. It is plain to see that there is a large difference between the theoretical and the actual profile; therefore, it is possible to conclude that the criterion based on overturning balance is not able, by itself, to explain the structural behavior of corbelled domes.

2.3. The modified corbelling analysis

The results obtained in the previous paragraph imply the need to include in the modeling of the structural behavior of the corbelled dome a kind of "collaboration" between the meridian wedges of the dome, that simulates horizontal actions along the parallel, produced by the engagement and friction between the blocks, or from the cohesion if mortar is present.
A new model, able to evaluate a cooperation among the blocks along the horizontal rings, is now proposed, in analogy with the membrane theory for thin domes that determines the stress both along the meridians (arches) and along the parallel (the laying of the blocks) (Timoshenko 1959, Flügge 1962). Such model is always located in the corbelling theory, that is based in considering only vertical forces and limit balance equations for overturning, under the already listed hypotheses. The difference with the previous model consists in considering meridian wedges of finite size (depending on the parameter ϕ, meridian wedge width) simulating a continuity, at least partially, along parallel, and therefore cooperation between infinitesimal meridian wedges. The cooperation between meridian wedges depends on the engagement and friction between the blocks and on the cohesion if there is mortar.
The assumption of meridian wedges with finite size implies the displacement of the overturning axis toward the center of the dome, from X to X ', Fig. 8B, thereby increasing the stabilizing moment (Ms) than overturning moment (Mr). This increase is progressively greater with increasing width ϕ of the meridian wedge. The angular variation ϕ of the meridian wedge conceptually expresses the extension of cooperation along the horizontal rings characterizing a spatial mechanical behavior. With reference to Fig. 8B, in the expression of overturning moment Mr and stabilizing moment Ms respectively the distances from the rotation axis of the gravity centers of the infinitesimal areas in the region OPMA and of the overall region PNM depend on width ϕ. For this reason, the balance equation, $Mr = Ms$, changes and is given by

$$\int_0^x [Y'(t)h(t)]\left(x\cos\frac{\varphi}{2} - t\right)\, t\, dt = \left\{\frac{Y'(x)h^2(x)x}{2}\right\}\left(\frac{h(x)}{3} + x - x\cos\frac{\varphi}{2}\right)$$

This equation is again an *integral* equation of Volterra of the second type, which can be reduced in *differential* form through 2 subsequent derivatives, implemented in Mathematica 6. Fig.10 show a comparison between real profiles and their better approximating theoretical limit curves respectively for the Syrian's dome (Fig.10A) and for the dome of the Puglia's "Trullo" (Fig.10B). In both cases, the angle ϕ that determines profiles better approximating to real domes is between 70 ° and 80 °.

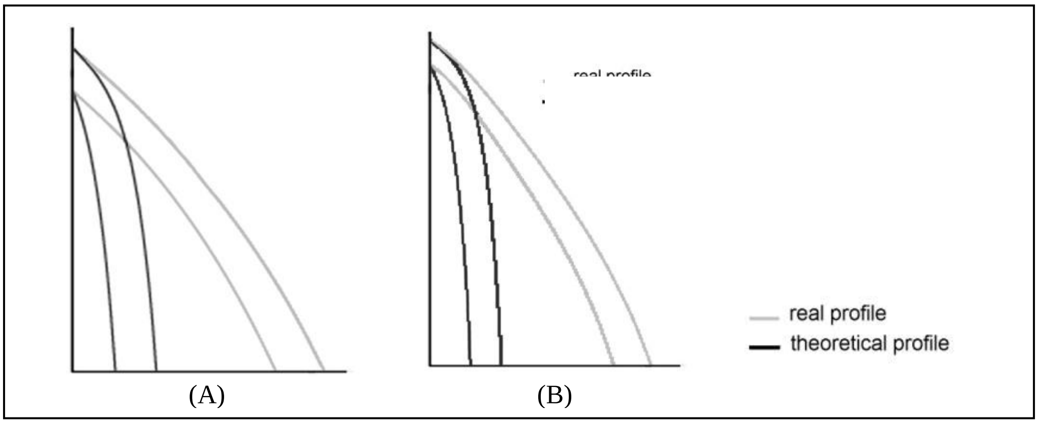


Figure 9 - Comparison between the real profiles and the theoretical profiles determined by the corbelling theory applied to an infinitesimal meridian wedge: (A) Syrian dome (B) "Trullo" of Puglia.

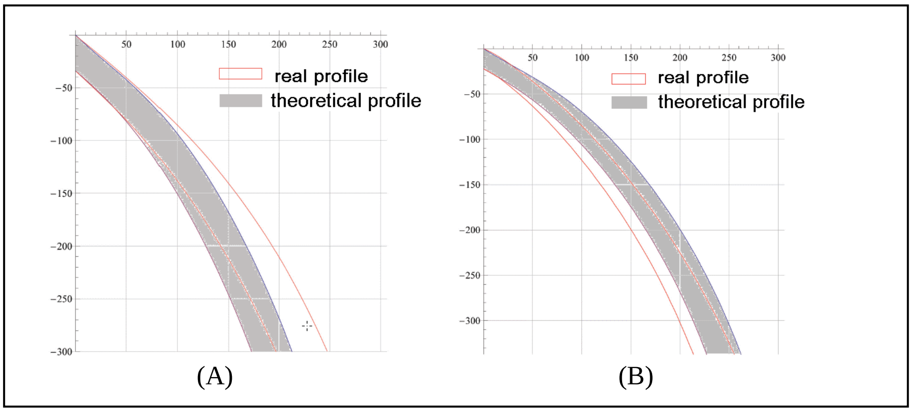


Figure 10 - Comparison between the real profile and the theoretical profile determined by the modified corbelling theory: (A) Syrian dome (ϕ =75.5°); (B) Apulia Trullo (ϕ =72.5°).

2.3. The FEM analysis

The "Straus7" software was employed on two case studies in order to carry out a numerical analysis by means of the finite element method (FEM). This software allows to perform linear elastic analyses using plate/shell finite elements, characterized by both membrane and bending behavior. The elements (called Quad4) are characterized by four nodes with six degrees of freedom for each node. Each element is characterized by a thickness that takes into account the real thickness of the domes, which varies with the height. The nodes at the springing of the domes were restrained with hinges. The results obtained by this analysis are shown in Fig.11 and Fig.12 respectively for Syrian dome and for Apulia Trullo [where the parallel and meridian stress, on all the elements, on the extrados surface (A, B) and the thrust line (determined by the evaluation of eccentricity of the internal forces) within the dome thickness (C) are shown].

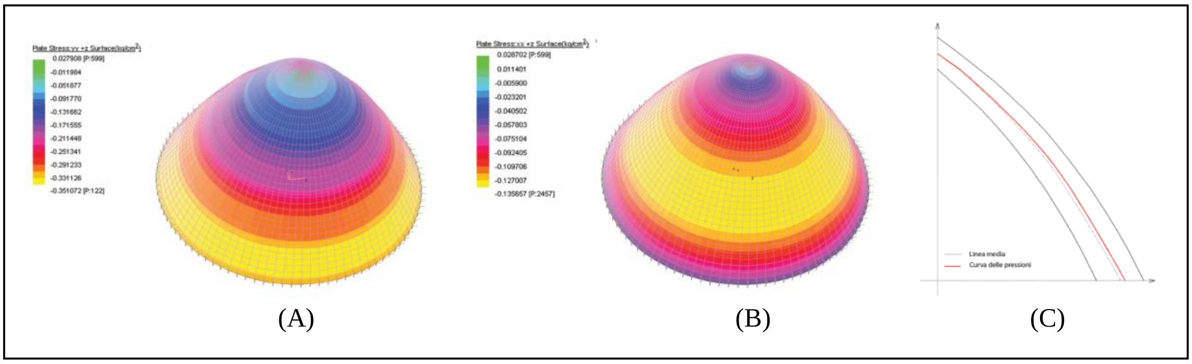


Figure 11 – Resuls of FEM analysis on Syriam dome.

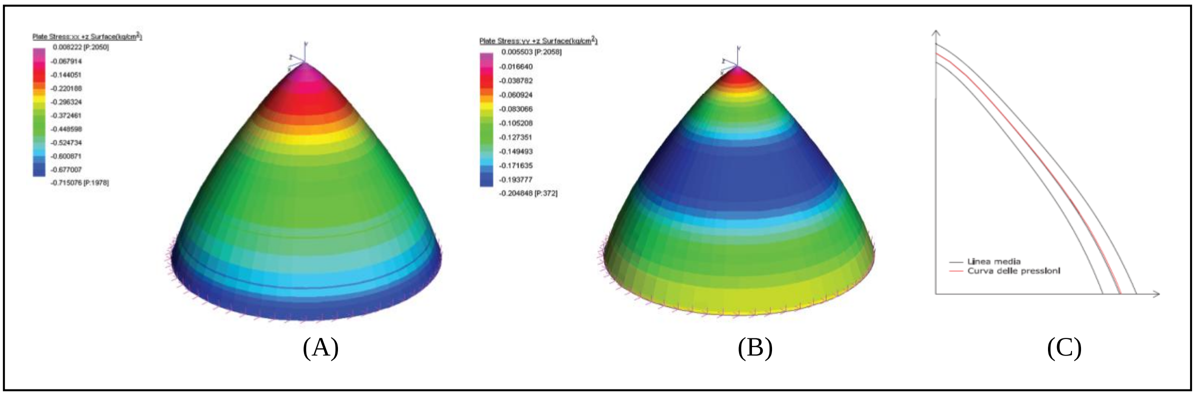


Figure 12 - Resuls of FEM analysis on Apulian Trullo.

It is possible to note that the FEM results are very similar for two case studies . In particular, the stress along the meridians are very low, and, their variation shows a progressive increase of compression in the direction of the springs (up to a maximum of 0.034 MPa for Syrian dome and 0.067 MPa for the Apulia Trullo). As for the stress along the parallel, all rings, at any height, are compressed. The stress values are very low, about equal at the key and at the springs, with a maximum (0.011 MPa for Syrian dome and 0.017 MPa for the Apulia Trullo) at the height corresponding to the half of the dome. The values and the distribution of the stress allow to exclude the presence of critical zones for the strength in particular, the absence of any tensile stress assures, from a mechanical point of view, the safety of the analyzed dome. The most interesting results are the thrust lines, that highlights the membrane behaviour of the domes, since the thrust lines practically coincides with the middle surface of the dome (Heyman 1977). This fact confirms the hypothesis of a spatial behavior of the domes and legitimizes the application of FEM model for no-tension materials too.

2.4. A basic test

With the aim of assessing the effective role of the friction to achieve a continuity in the horizontal stone(brick)-work, a basic test was carried out. Using as blocks in-scale bricks (10x5x2.5 cm), some of them were superimposed giving a jut of 2cm. Three different structural layouts were compared: a sequence built on a single base block, then a sequence built on a straight row of some blocks, finally the building of a segment of dome on a curvilinear arrangement. The surprising results highlighted that (Fig. 13): the single brick row collapses with the laying of the fifth block, the straight short row layout collapses with the sixth block, while the circular segment of dome reaches its peak with no problems. It is possible to deduce that an effective friction arises between the faces of the bricks (due to the contact forces both horizontal and lateral), but its effectiveness and extent depends on the shape of the brickwork (straight or circular). Lastly the circular arrangement shifts the overturning axe in a more favorable position. The last layout perfectly simulates the shape of very common fragments of domes standing, as ruins, after partial collapses both with regard to earthen and dry stone buildings (Fig.13).

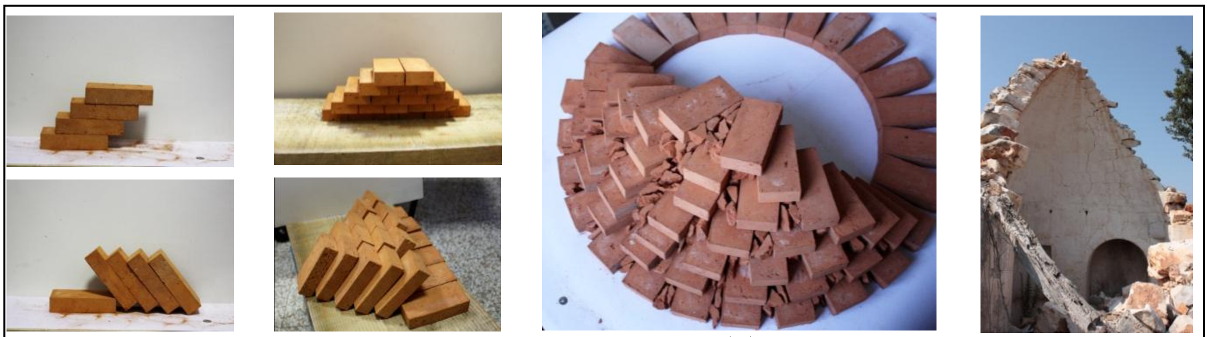

(A) (B) (C) (D)

Figure 13 – Basic test: (A) single brick sequence; (B) straight row brickwork; (C) circular brickwork. (D) Standing fragment of a real dome.

3. CONCLUSION

The present work demonstrates, by analyzing two real cases, that the model based on the corbelling theory cannot interpret structural behavior of the corbelled domes, both built by earthen bricks and by dry stones: in fact an infinitesimal meridian wedge of dome cannot guarantee the overturning balance from the given geometry.

According to such considerations the model based on the corbelling theory was modified in order to assume the structural collaboration that arises along the horizontal layers of the dome. Such collaboration, that modify the structural behavior, is due to the friction that develops between the blocks and to the cohesion derived from the use of mortar. As regards the analyzed cases, the “in situ” surveys and the interviews with the masons experts of building traditional techniques highlighted that, in both cases, the techniques employed in the domes construction are really designed to achieve the better continuity along the parallels. In the Syrian case the earth material allows to create a sort of

conglomerate equipped of cohesion, both during the building process and at its conclusion (both mortar and bricks are made by the same earth, able to join completely when wet). On the other hand, in the Apulian Trulli, but extensively in dry-stone constructions, the building technique by which the horizontal rings are made is based on minimizing the presence of empty spaces or constructive discontinuities. This is achieved both through squared ashlars (placed in a regular laying surface) and through the foresight to put some wedge around the perimeter of the ring and into the gaps remaining between two adjacent ashlars. The aim of such procedure is to create horizontal rings made by stones but able to endure a compression along the parallels, so hindering any attempt of overturning inwards. Such phenomenon can completely explicate when the ring is enclosed, but, thanks to the friction originating among the lateral surfaces of the ashlars, it exists even if the ring is not completed and increases with the increase of the number of the ashlars, as the basic test demonstrated.
The proposed model estimates the entity of the actions along the parallels by imposing a solidarity (a kind of partial monolithism due to the collaboration) among the meridian wedges of the dome. These collaboration is measured by an angle which defines the finite width of the meridian wedge of dome necessary in order to guarantee the balance against overturning. A further confirmation of the suitability of a 3D model can be deduced from FEM analysis. Such analysis suggests the hypothesis of a structural behavior evaluable through the membrane theory and, for the investigated domes, strictly confirmed by their particular shape, that determines a healthy compressive stress in the material, and by the building techniques able to use such stress for stability.

REFERENCES

Barbi, L. (2000). "Analisi delle caratteristiche tecnico-costruttive della Tomba dei Carri". *L'architettura Funeraria a Populonia tra IX e VI sec. a. C.. Atti del Convegno di Populonia*, 1997, Firenze.

Benvenuto, E., Corradi, M. (1988). "La statica delle false volte", *Nel cinquantenario della Facoltà di Architettura di Napoli, Franco Jossa e la sua opera,* Napoli.

Cavanagh, W. G., Laxton, R. R. (1981). "The Structural Mechanics of the Mycenaean Tholos Tombs." *The Annual of the British School of Athens,* 76, pp. 109-140.

Como, M. T. (2005). "L'architettura delle tholoi micenee: analisi degli aspetti compositivi, costruttivi e statici", Doctoral Dissertation, Università degli Studî Suor Orsola Benincasa, Napoli.

Como, M. T. (2006). "Analysis of the Statics of the Mycenaean Tholoi." *Proc of The Second International Congress on Construction History,*Vol.1, Cambridge.

Besenval, R. (1984). "Technologie de la voute dans l'Orient Ancien." *Recherche sur le Civilisations*, Paris.

Fábrega, A. (2003). "Estudi mecanical de la falsa cupola." *SCM Notícies* n.19, p.12-18.

Flügge, W. (1962). "Stresses in shells." Springer-Verlag.

Juvanec, B. (2001). "Six Thousand Years, and more, of Corbelling, Age of stone shelters." *UNESCO/ICOMOS Congress I,* Paris.

Juvanec, B. (2009). "Corbelling of the Mediterranean." *Earthen Domes and Habitats,* S. Mecca and L. Dipasquale, ETS, Pisa.

Moramarco, V. (2009). "I trulli di Alberobello. Lettura costruttiva e indagine static." M.S. thesis, Università degli Studi di Firenze.

Mylonas, G. E. (1966). "Mycenae and the Mycenaean Age", Princeton.

Palyvou, C. (2009). "Prehistoric dome architecture in the Aegean". *Earthen Domes and Habitats.* S. Mecca and L. Dipasquale, ETS, Pisa.

Paglini, M. (2008). "Analisi costruttiva e studio del comportamento strutturale di cupole ad assise orizzontali", Doctoral Dissertation, University of Florence

Paglini, M., Rovero, L., Tonietti, U. (2009). "Constructive and structural analysis: identification of the structure." *Earthen Domes and Habitats.* S. Mecca and L. Dipasquale, ETS, Pisa.

Paglini, M., Rovero, L., Tonietti, U. (2009). "Statical analysis of the earthen corbelled domes." *Earthen Domes and Habitats.* S. Mecca and L. Dipasquale, ETS, Pisa.

Pelon, O. (1976). "Tholoi, tumuli et cercles funéraires", Paris.

Santillo R., Fritzell Santillo B. (1984), "The Construction and Structural Behaviour of the Mycenaenan Tholos Tombs", OA, XV, 44-52.

Rohlfs, G. (1963). "Primitive costruzioni a cupola in Europa", Leo S. Olschki, Firenze.

Timoshenko, S. (1959). "Theory of plates and shells", McGraw-Hill Book Company.

Wace, A.J.B. (1921). "The Tholos Tombs", BSA 25, 1921-23, pp.283ss.