Arches, Domes and Vaults in the History of Architecture

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The first real domes arise and develop with the Romans. The first important example is the dome of the Octagonal Room in the Domus Aurea (I cent.).
But the Pantheon (II cent. inspired by the Domus Aurea), more than 40 meters in diameter, is the true original example of the large domes that will be built in the Western world. The next advance takes place several centuries later with Hagia Sofia (VI cent.). We must wait for the dawn of the Renaissance to have a breakthrough in structural design with the dome of S. Maria del Fiore (XIV cent.) and then with the dome of St. Peter's, both clearly inspired by the Gothic. After St. Peter's, the seamless integration between structure and architecture and the momentum of building domes seems to vanish.

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1. Foreword

The arch has probably been the most important innovation in the history of architecture, transforming, thanks to the curvature associated with the thrust at the springs, bending moments in compression forces, even if a certain bending strength is indispensable to maintain a stable shape.

Beams, catenaries and arches were the basic monodimensional elements that marked the development of architecture and structures.

Beams

The beam is the simplest element, but its simplicity does not combine with the most favourable structural behaviour.

The bearing capacity of beams is provided mainly by bending moments, so that equilibrium is assured only if compression and tension stresses can be developed on the opposite part of each section.

This behaviour involves two main requisites: 1. the material must be resistant to both compression and tension stresses; 2. the spans cannot be too long because the arm of the resistant bending moment is small, being only a fraction of the section's thickness; consequently stresses easily become incompatible with the strength of ancient materials.

For a long time wood was the material that best satisfied these requirements. However, its limited durability and frequent destruction by fire have created serious problems.

On the other hand, stone beams and lintels cannot be adopted for long spans, on account of their limited and fragile tensile resistance. And there are also difficulties in quarrying large monolithic pieces.

To reduce the load on the lintels the overhanging structure is often organised so as to load most of the stress directly onto the springers, as in the Lion's gate at Mycenae (fig. 1). It has been only in the last two centuries that the production of new materials (steel, reinforced and prestressed concrete, etc.) has allowed the creation of beams with exceptional spans, transforming them into the main element of modern architecture.

Fig.1. The Lion's gate.

Catenaries

Catenaries are undoubtedly more rational structural elements: the horizontal reaction at the ends provides a supplementary force that allows the material to act with a single, uniform tension stress; the specific characteristics of each material are then exploited to their best advantage and the resulting bearing capacity, thanks to the increased arm, which is now represented by the sag, is much greater than that of beams.

Unfortunately, the high deformability of cables and ropes can impair the stability of their shape, limiting their use (fig. 2). Here, too, the development of special steel and modern techniques to overcome these difficulties has permitted the construction of large structures, the most outstanding examples being suspension bridges.

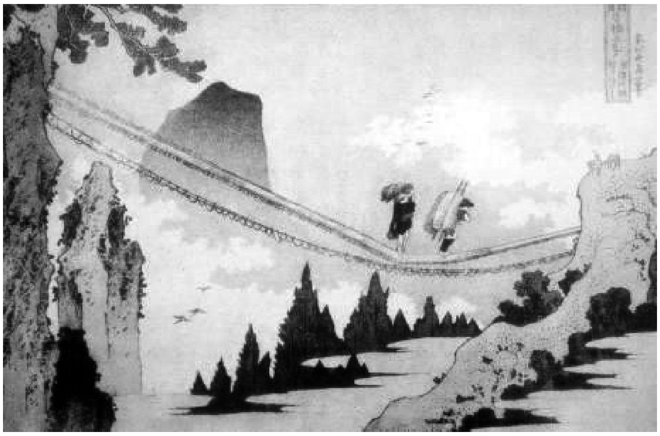

Fig. 2. A suspended bridge.

Arches

The arch is the element that combines the advantage of the beam (stiffness) and the catenary (lower stresses, on account of the large arm provided by the thrust at the springers). Moreover, as the behaviour of the catenary is inverted (tensions become compression stresses) continuity is no longer required and it is sufficient to ensure contrast between independent elements: stone blocks that contrast each other and are easy to quarry are thus the chief material used in arches.

Displacements of the springers, often due to soil settlement or deformation of pillars, reduce the efficiency of the thrust and are the most frequent problem with this kind of structure. To prevent this phenomenon, wood or steel chains are frequently used.

It is interesting to observe that the arch behaviour is magnified by the curvature of the structure but that this behaviour can be realised also in a very flat arch, or even in a beam, if the support does not allow any relative movement. In this case, a horizontal force (the thrust) is produced which, composed with vertical loads, makes possible the flow of the stresses along curved ideal lines (arch effect) produced inside the thickness of the beam. The architraves (made of brick) in the Octagonal Room in the Domus Aurea are an exceptional example of the arch effect (fig. 3).

The weakest points in the dry block arches are the joints, where the shear stresses (parallel to the joints) can't exceed the friction strength.

To avoid these problems, the joints in Roman arches and vaults are placed perpendicular to the geometrical axis; which is close to the line of the resulting compression force.

In the Khmer arches and vaulted spaces (as often occurs in Asian architecture), the blocks are cut in a different way so that the joints are horizontal.

The consequence is that the direction of the resulting compression force is inclined with respect to the joint with a possible risk of sliding, exceeding the friction resistance.

To prevent this risk and to reduce the inclination of the forces, heavy loads are often applied with important architectural effects, as in the Bayon Gate in Angkor (fig. 4).

Vaults, domes and towers

Spatial vaults (usually with circular, square or polygonal bases) are a development of the arch concept. Their particularly satisfactory behaviour is due to the double curvature and mainly to the hoop effect of horizontal rings.

The cooperation between the forces that flow through the meridians and the parallels allows exceptional resistance to be achieved.

This gives the possibility of choosing different architectural shapes even when these are not the most rational and efficient from a structural point of view, as in the case of the Bulbous domes in Islamic countries (fig. 5).

Similar to the arches, in dry stone structures, the position of the joints influence the shape. In Asian architecture where the joints are usually horizontal, domes are transformed in a kind of tower due to the necessity of increasing the vertical forces in order to reduce the shear forces and therefore the risk of sliding between joints.

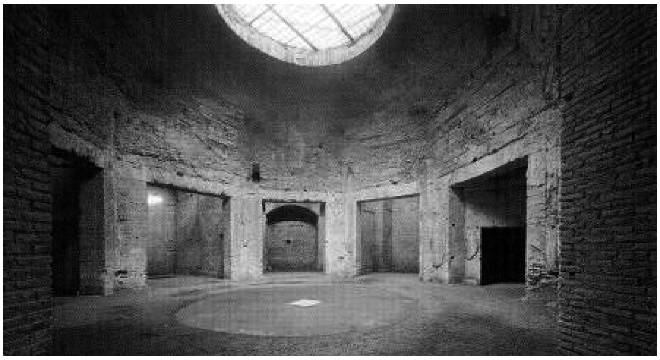

Fig. 3. The architrave of the Octagonal Hall in the Domus Aurea.

Fig. 4. The South Gate in Angkor Thom.

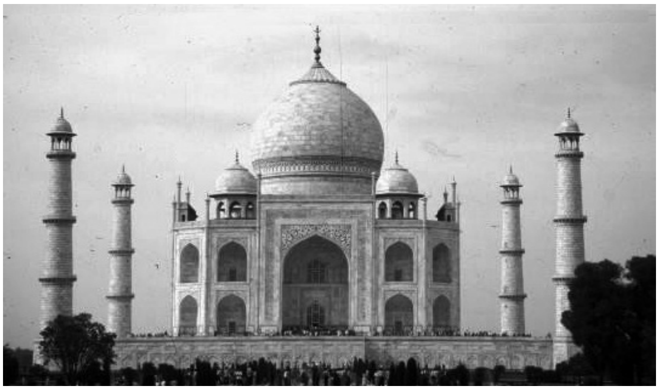

Fig. 5. The Taj Mahal.

2. The Pantheon

The Pantheon (built under the Emperor Hadrian in the 2nd century AD, fig. 6) is apparently a very simple structure made of a cylinder (the Rotunda) and a hemispheric dome of the same diameter (around 43 m).

However, if we look at it in detail, it is much more complex, full of intuitions and innovations. The relationship between the cylinder and the dome appears very different if appreciated from outside or from inside.

From inside the dome it is clearly a hemisphere where the meridians spring vertically from the cylinder itself. From the outside, the cylinder appears higher than from inside and the dome emerges from the cornice with a flatter shape. It is interesting to note that the “steps” visible on the extrados of the dome are not an architectural choice but the consequence of the technique of pouring the concrete in subsequent rings.

From outside, the cylinder appears as a big brick wall, containing within its thickness a series of arches which inside correspond to niches and empty spaces.

Probably the builders of the Pantheon were influenced by the Coliseum which, even if it appears completely different in its structure, in reality is more similar to the Rotunda than one would expect. The photomontage of fig. 7 gives an idea of that.

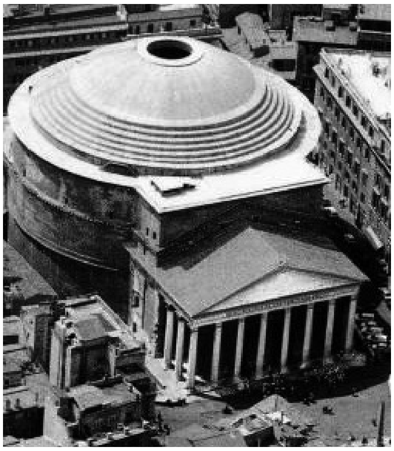

Fig. 6. The Pantheon in Rome.

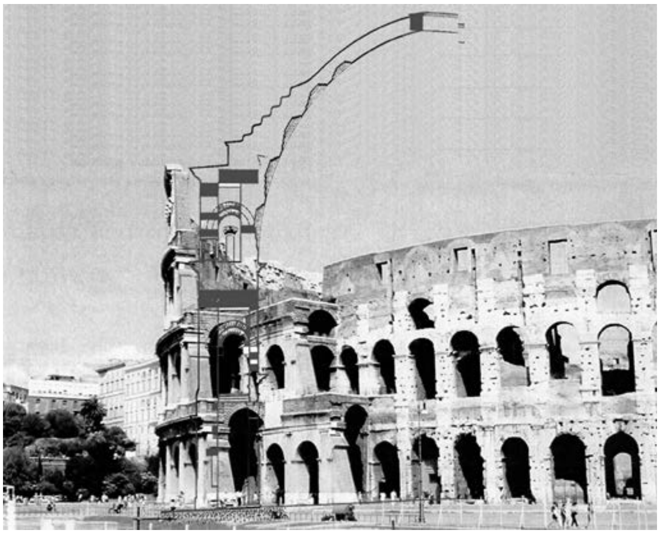

Fig. 7. Photomontage of a section of the Pantheon and the Coliseum.

3. Hagia Sophia

Hagia Sophia (built in the present shape under the Emperor Justinian in the 6th century, fig. 8) chronologically is the second biggest dome in the history of architecture and got its inspiration from the Pantheon. However, Hagia Sophia features some important differences related to the fact that the dome is supported by four huge pillars placed on the corners of an ideal square base (32 meters). Two problems arise:

— how to resist the circumferential forces at the border of the dome;

— how to transfer the vertical forces from the meridians to the pillars.

Fig. 8. Hagia Sophia in Istanbul.

Fig. 9. Dome, arches and pendentives in Hagia Sophia.

The solution of the first problem was the introduction of hemidomes and abutments to balance the thrusts while "pendentives" on the four corners, associated with arches, have solved the second issue, allowing the forces to flow from the top to the ground (fig. 9).

These innovations turned out to be very important; most of the following domes are inspired on these principles.

Hagia Sophia suffered from several earthquakes. The first one took place just as the construction was completed and produced large collapses. The cupola was immediately rebuilt.

The second major collapse, in the 10th century, involved the Western portion of the building with the arch and a quadrant of about 140° of the dome. This part was rebuilt in the following years.

The third collapse took place on the 14th century and produced a collapse, similar to the former one, on the opposite side.

At the present time, the dome shows four lines of discontinuity following the meridians, large deformations on the ribs, marking the connections between original and rebuilt portions. Different analyses show that critic situations arise when the earthquake acts in the direction of the abutments: concentration of stresses is produced on the pendentives and the supporting arch of the dome. Mathematical models show the deformation of the key of the arch in agreement with what is the actual experienced behaviour.

4. The temples of Angkor

Angkor is an extraordinary site where ancient temples (built between the 9th and 13th century AD) emerge from the tropical forest that through the centuries has gradually overgrown them. Angkor Wat, built in the 12th century, is the masterpiece of Khmer art (fig. 10).

It is a three-tiered mountain temple and is a physical representation of the Hindu cosmology. Five central towers represent the peaks of Mount Meru, the Olympus of Hindu gods and the centre of the universe. The outer walls represent the mountains at the edge of the world, and the moat of the oceans beyond.

The temples have a typical tower shape that is the consequence not only of the Khmer architecture but also (and perhaps mainly) of the need for stabilising the structure, built with sandstone blocks and horizontal joints (as already mentioned in paragraph 1).

The main causes of damage and decay are related to the stone deterioration and to the effect of trees (in some cases the roots, penetrating into the joints, have progressively enlarged the joints, displacing the blocks, until they collapse, fig. 11). Soil settlement related to the change in the underground water table is another important factor: the site of Angkor was totally abandoned from the 12th to the 19th century, when the Khmer Kingdom was defeated by the Siams.

Fig. 10. The Temple of Angkor Wat.

Fig. 11. The destructive effects of the roots of the trees in the Temples of Angkor.

5. The Basilica of St. Francis of Assisi

The Basilica of St. Francis of Assisi was built in the 13th century.

It suffered several earthquakes, but none produced damage as great as that which hit the Basilica on September 26, 1997 causing the collapse of two vaults (fig. 12), large cracks everywhere and the failure of the tympanum of one transept.

Urgent measures, to prevent large collapses, were required on the vaults. After having installed a provisional "flying bridge" suspended from the roof, some synthetic fibre strips were applied to the vaults over the cracks, and a system of wires and springs to suspend the vaults to the roof was applied as well. The springs were inserted to maintain the force of the design value, independent from thermal effects, also cutting the transmission of the vibrations. Different mathematical models were prepared to study the structural behaviours under the effect of seismic forces (fig. 13).

The mathematical models show that the earthquake produce large tensions on the ribs of the vaults, coherently with the collapse mechanism of the "hinge" created in the middle of the rib at the moment of the collapse, which is clearly visible.

The first work has been the reconstruction of the collapsed vaults by using as many as possible of the original salvaged bricks. With reference to the large cracks and the permanent deformations that generally affected all the vaults, it was decided to build, over the extrados, a net of ribs made of timber strips covered with a fabric of aramidic fibres and epoxy resin (fig. 14). Those elements were previously tested in a specialised testing laboratory. To reduce the deformability a system of tie bars and springs connecting the vaults and the roof was built as well.

Fig. 12. Interior view of the Basilica of St. Francis of Assisi.

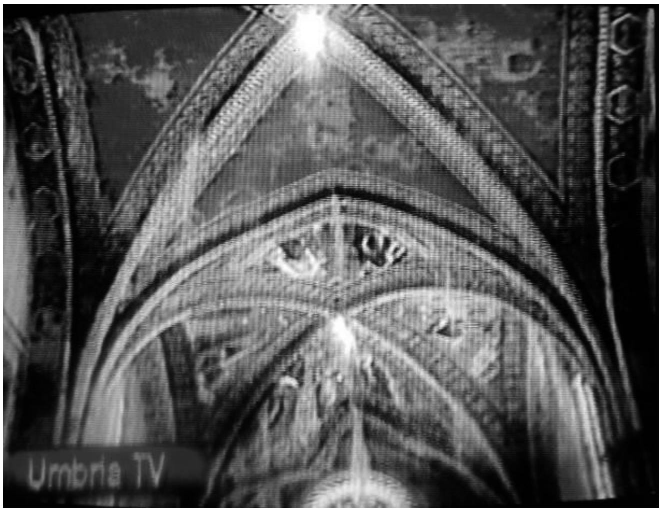

Fig. 13. The instant where the collapse began (the broken rib is visible).

Fig. 14. The ribs on the extrados of the vaults.

6. S. Maria del Fiore

The dome of the Church of S. Maria del Fiore (Brunelleschi, 15th century – fig. 15) is the first example of a big dome with a double shell on an octagonal plan. The dome, having a diameter 43 m similar to the Pantheon, was inspired by Gothic vaults. In order to reduce the thrust the shape is ogival and to reduce the weight the main bearing structure is made of 8 principal ribs (or spurs) in the corners and 16 supplementary ribs in the middle of the webs (or segments of the shells). The circumferential connection is ensured by 4 stone ribs (a kind of "chains of stone" reinforced with steel clamps) and a wooden chain; in addition small horizontal arches improve the connection between the corner ribs and the adjacent ribs on the webs.

One of the main issues that Brunelleschi had to solve was how to build the dome without scaffolding, which would have been too big and too heavy.

Most likely in this choice Brunelleschi was guided not only by the Pantheon and Domus Aurea (the "oculus" had shown that the equilibrium was possible without extending the structure up to the top), but also by vaults and domes constructed in Persia (it is possible that Brunelleschi made a trip there) where, following an old tradition, the stability during the construction is ensured by a precise study of the layout shape and interlining of the bricks (herringbone etc.), in such a way to make possible the development of an horizontal "arch effect."

The octagonal shape of the dome, congruent with the drum and the plan of the church underneath, demonstrates that, thanks to the "stone chains" or supplementary steel chains tensile strength, protects it from seismic events. Some small cracks visible on the shells, in the zones over the windows, appear to be related more to the construction phase than to seismic effects (earthquakes are very rare in Florence).

The octagonal base of the dome is made of four huge pillars on four sides and four arches in between so that the supporting structure is stiff (fig. 16), and even more stiffened by the connection with small hemidomes, probably inspired by Hagia Sophia.

On the top of the dome there is a lantern; this implies that differently from the Pantheon the meridians arrive at the edge-ring of the oculus with a smaller inclination with respect to the vertical line, necessary to support the weight of the lantern.

Fig. 15. The Church of S. Maria del Fiore.

Fig. 16. Interior view of S. Maria del Fiore.

7. St. Peter

The dome of St. Peter (Michelangelo 16th century, fig. 17) has a diameter similar to the Pantheon and S. Maria del Fiore (43 m) and, like the latter, has been deeply influenced by the Gothic conception. Nevertheless a general harmony and classical inspiration is evident.

The project of the St. Peter's dome had a number of difficulties before arriving at Michelangelo's final design.

A significant difference between the domes of Brunelleschi and Michelangelo is in the shape of the drum, that here is circular, even if the dome is made with 16 ribs. The circumferential stone chains of S. Maria del Fiore, in St. Peter are replaced with steel chains.

From seismic point of view there are certain analogies between the structural scheme of Hagia Sophia and S. Peter. As in both cases the forces are obliged to flow from the circular plane of the dome to the four columns placed on the corners of an ideal square, through four arches and relative pendentives.

The substantial difference however is that in St. Peter there is a strong drum whilst in Hagia Sophia, as we have seen, not only is the drum absent, but the base of the dome is weakened by a series of windows.

Even if St. Peter's appears to be strong enough against earthquakes, it is likely, however, that the earthquake that hit Rome in 1703, and which acted on a structure that had suffered from shrinkage and viscous phenomena (due to the rush of completing the construction) and probably from some differential settlement (in relation to the presence of not homogeneous ancient underground structures), has contributed to the cracks that caused such considerable alarm to Pope Benedict XIV in the middle of the eighteen century that he had the dome reinforced with circular steel chains.

Fig. 17. The dome of St. Peter.

8. Arches and domes after St. Peter's

St. Peter's (16th century) represents the last exceptional dome ever built, having a diameter of around 42 mt., as already mentioned, and is similar to the Pantheon, Hagia Sophia and S. Maria del Fiore.

In the following centuries these feats have never been achieved again, even if the dome of St. Paul in London (17th century) (fig. 18) and the dome of the Pantheon in Paris (18th century), both built with three vaults (diameter around 34 mt.), represent interesting solutions. However, the strong identification between structure and architecture that has characterised the previous domes, appears to be lost: it is only the intermediate shell which provides the structural bearing capacity, while the inner and external vaults have mainly a decorative purpose.

The development of new techniques and technologies has transformed the static conception of the vaults, and steel begins to be preferred to masonry even if these steel structures are often hidden under a masonry casing; this is the case of the dome of the Capital in Washington built in the 19th century, where a steel reticulated dome is covered with blocks of stones.

The reconstruction at the end of the 20th century of the Reichstadt dome in Berlin, destroyed during the last world war, is an interesting example of the use of steel that doesn't need to be hidden, but on the contrary, shows all its exceptional static and architectonic possibilities.

Last, but not the least, we would like to mention the Millennium Dome built to celebrate the passage between the 20th and the 21st century, so huge that today nobody exactly knows how to utilise it. Probably it will be dismantled.

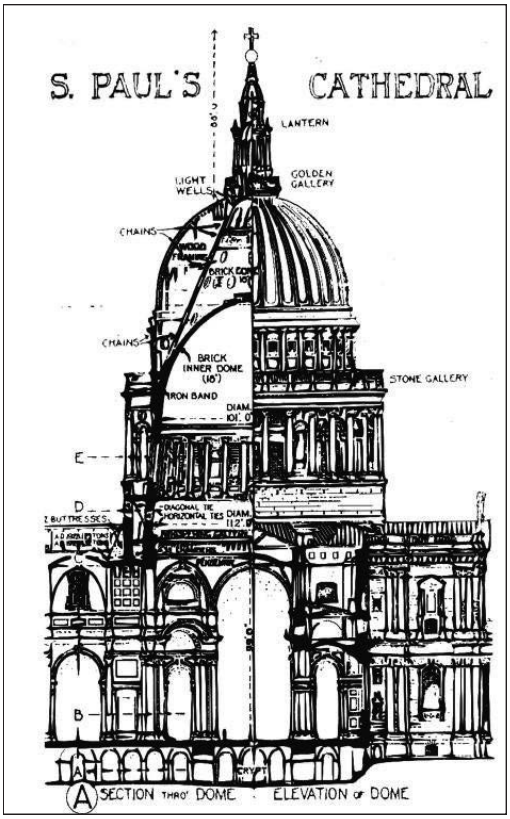

Fig. 18. St. Paul Cathedral in London.

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