Brunelleschi and the Dome

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The dome of Santa Maria del Fiore is unique of its kind. For centuries it has been the subject of research by scholars who tried to ferret out the 'secret' of its construction. The studies permitted:

- Knowledge of 'the object' with clarification of the areas to be explored.
- Knowledge of the various theories hypothesized up to now.
- Elaboration of data that consists of a study of the electrical characteristics of the materials constituting the sections, and precisely the speed of propagation of the electromagnetic waves and relative electrical permittivity (dielectric constant) and depths reached.
- Interpretation of data and creation of radar-grams, that is, interpretive graphics.
- Comparison between the results thus obtained and present knowledge of the dome.
- Creation of an experimental model of a portion of the vela, with the same qualities as the real one, made with the same materials, and placed according to information that came out of the studies. The comparison between the results of the study carried out on the model and on the dome itself allowed for the making of precise affirmations on the typology of the structure.

Keywords: *Brunelleschi, S. Maria del Fiore, Dome, Geometry, Survey.*

1. Brunelleschi and the Dome

Filippo Brunelleschi was born in Florence in 1377. His main profession was architecture, but he was also a goldsmith, sculptor and mathematician. His major work is the Dome of the Cathedral of Santa Maria del Fiore in Florence. Brunelleschi lived near the Duomo Square; since his childhood, he had seen masons working on the construction site of the Duomo. He had a chance to observe the machines used to lift marble and stone, and was fascinated by them. In fact, he was very interested in mechanics too, and so became apprentice in a workshop owned by a goldsmith, Benincasa Lotti, a family friend. His decision to become a goldsmith happened because, in those times, goldsmiths not only made jewelry but also built clocks: consequently their abilities embraced several fields of knowledge. In order to fuse gold, copper and bronze, these substances had to mix with other elements such as lead and sulfur; the molds into which these materials were poured, were made of clay; thus Brunelleschi had to have some knowledge of chemistry. This knowledge turned out to be very important for the construction of the Dome: it allowed him to understand in depth such materials as mortar, stone, clay and metals with which brackets were forged to keep the wall components joined. To build clocks, Brunelleschi had to know mechanics as well. He built many clocks; the only surviving example can be admired in the Palazzo dei Vicari di Scarperia, near Florence. These skills were very useful too: they allowed him to design machines used to hoist the materials. These wonderful machines remained in the construction site of the Cathedral after Brunelleschi's death in 1446, and were used afterwards, especially to build the Lantern, also designed by Brunelleschi, but assembled by his step-son Buggiano. In 1472, Verrocchio built a bronze sphere, which he placed on top of the Lantern, using the machines designed by Brunelleschi, which were still at the Cathedral's site. Among his workshop trainees, who helped him with this operation, was a young man from Vinci, named Leonardo. He took notes and designed machines: and so Brunelleschi was the inspiration for Leonardo to build the machines for which he became

Fig. 1. The Dome view from the corner of Calzaioli Street.

Fig. 2. View from below the inner Dome. (Photo by A.R. Rothschild)

famous. Brunelleschi was also an expert mathematician and was the first to determine the geometrical laws of linear perspective. He proved his knowledge by crafting an experiment in perspective. He painted the Cathedral of Florence on a drawing board, using the geometrical rules of perspective. On the board, in correspondence to what is called the "vanishing point", he pierced a little hole, from which one can see the image drawn on the drawing board reflected in a mirror placed between the observer and the Baptistery of Florence. The onlooker, standing at the Cathedral's door, thinks he sees the real Baptistery: in reality he sees only its image reflected in the mirror. Unfortunately, this tablet has been lost. But one can observe his mathematical bravura in the application of the laws of perspective, observing the Trinity that was painted in 1425 by his friend Masaccio, on the right wall of Santa Maria Novella. It was almost certainly Brunelleschi who executed the perspective framework of this fresco; one can see that the geometrical principles that he used are quite rigorous, obtaining a great scenographic effect through its sense of depth. In 1401 he participated in a competition, the winner of which would execute the Baptistery doors. The seven candidates had to complete a tile within a year. At the end, only two were left: Brunelleschi and a goldsmith who was then unknown: Lorenzo Ghiberti. Each of them executed a tile on the theme, "The Sacrifice of Isaac."

The 34 members of the jury did not know whom to choose and, as usual, they split into two factions. In the end, Lorenzo Ghiberti won, but, in Brunelleschi's opinion, there were some irregularities and preferential treatment in his favor. Many years later, Brunelleschi wreaked his revenge on Lorenzo. Between the two, a great rivalry had begun, which would last all their life. Disappointed with the outcome of the competition, after a while Brunelleschi gave his tile as a gift to Cosimo de' Medici and left for Rome with his dear friend Donatello, the famous sculptor, as recounted in Brunelleschi's biography written by Antonio di Tuccio Manetti. He did not reveal to anyone the reason for his trip; he actually knew that the construction of the Dome would begin in a few years. He wanted to be ready for that moment. The city of Rome was an immense archeological field of buildings made by the ancient Romans, partly visible and partly buried. Brunelleschi wanted to discover the secrets of these buildings. For this reason, he went to Rome several times in the period between 1406 and 1416. Moreover, the dome of the Pantheon, with dimensions similar to the one he would have had to erect, was located in Rome. The Pantheon had been built under the emperor Hadrian between 118 and 128 A.D. Its dome has the shape of a semi-sphere with a diameter of 43.20 m. Its dimensions are thus comparable to those of the Dome he would design; in fact, its basis is an octagon with an internal diameter of approximately 45.00 m. The presence of the large open "oculus" in the central point of the Pantheon could have shown the designer that even a larger vault could sustain itself at every stage of construction. Nonetheless, while the base of the Pantheon is a circumference, that of the Dome is an almost regular octagon. The circumference is a perfect shape; the octagon is not. The circumference does not have corners; the octagon does have them. A dome with circular base can be built more easily than one having a polygon as a base; and this is really one of the greatest difficulties in the process of building a dome The choice of an octagonal dome for the Cathedral of Florence was taken from its tradition; in fact, when Arnolfo chose the shape of the Dome, he used as an example the Baptistery of Florence, where an octagonal dome is found within. The dome of the Baptistery has the same shape as that of Santa Maria del Fiore, but it is much smaller, with a diameter measuring about 28.00 m.

In August 1418, the Opera of the Duomo set up a competition to appoint someone for the construction of the Dome. Brunelleschi, now back from Rome on a permanent basis, entered in the competition. Seventeen plans were submitted. After several vicissitudes, the jury concentrated on two plans: one by Brunelleschi and the other by Ghiberti. The story of the contest of 17 years earlier for the Baptistery was repeating itself! Again, the jury could not make a decision. Both the contestants had built a plan for the Dome in brick; the one by Brunelleschi had been built with bricks and limestone. The magnificent construction site opened its doors the day after the draft of the 1420 "apparatus," where it was explained how the drum of Santa Maria del Fiore was to be closed, based on the model made of bricks and limestone built by the designer in the immediate vicinity of the cathedral. The method is very simple: the brick walls are built by laying them on the centering. After the construction of the dome or the arch is finished, they can be dismantled: the brick structure is able to support itself! This technique had been used since ancient times. However, when a dome with circular base is built, there is no need of centering; in fact, in this instance, it is sufficient to build concentric rings of bricks. These bricks hold themselves up and the dome, as it grows, sustains itself: it is self supporting. The geometrical control of the laying of the bricks is done with a wooden stick, or with an iron thread, one end of which enters into the center of the sphere. This stick also functions as a guide to determine the inclination of the bricks. When the inclination of the bricks, placed along the parallels of the sphere, becomes disproportionate, they could, until the mortar hardens, slip down. To avoid this sliding, some bricks, at regular intervals, are placed *'a coltello,'* that is, vertically, with their longer side perpendicular to the parallels of the sphere; by so doing, they block the bricks positioned along the parallels. This technique is called "herring bone," because the bricks are placed like a fishbone. It is a particularly ingenious method, used by the Byzantines, who probably had learned it from the ancient Romans. Brunelleschi may have seen some Roman buildings that were built with this technique.

The "herring bone" was used, up until Brunelleschi's times, only for self-supporting domes; this was another reason why, during discussions with the examiners before the construction, the designer was advised to change the octagonal base to a circular one. In many Roman buildings, which Brunelleschi examined during his stay in Rome, the initial polygonal shape of the dome was modified to a circular one, as it rose and narrowed. Examples of this technique could not only be admired in the Domus Aurea, but also on the Domus Augustana, in Caracalla's thermal baths and in the temple of Minerva Medica. However, the members of the Opera del Duomo did not accept this, both because of the difficulty inherent in this change, and to respect tradition; moreover, it would have been a complex task to set up a circular-based dome on the already existing octagonal drum. In octagonal-based domes, everything is more complicated, in part because they do not have a uniform distribution of stress as do self-supporting domes, by virtue of their symmetrical radius. If the dimensions of the Dome had been those of Arnolfo's plan, everything would have been easier: centering could have been used, and its lesser weight would cause less stress. But in order to build the Cathedral's Dome it would have been necessary to build a wooden structure approximately 32.00 m tall, with its drum at a height of 55.00 m above the ground. Moreover, the wooden centering had to support a massive weight for many years (the Dome weighs about 29,000 tons and its construction lasted sixteen years, from 1420 to 1436).

The task seemed impossible; moreover, this structure would have cost a great deal. Finally, in 1420, the jury decided to nominate Brunelleschi and Ghiberti as "master builders" of the Dome. The Florentine people, knowing about the rivalry between the two, did not trust them; thus two other master builders were added: Battista d'Antonio and Giovanni da Prato. Initially Brunelleschi became angry at the presence of these two, whom he considered intruders, and he wanted to quit. Then he thought it over: this was, after all, a victory for him, because the plan to follow was his; in fact, he alone was credited as "inventor." The Florentines chose Brunelleschi's plan because it was the only one that could have been accomplished. They understood that it was the only plan that allowed the construction of the Dome; it was also the cheapest, because it did not include the construction of expensive wooden centering: a highly appropriate motive for the Florentines! Soon Brunelleschi was to rid himself of Lorenzo, by demonstrating his incompetence in architecture.

The winners of the competition on April 1420 presented, for the building of the Dome, a program that was later updated, both in 1422 and 1426. In this, the technical characteristics of the Dome were explained; the document specified as salient points the modalities of construction of the vault *majoris cupole*. However, as mentioned, Brunelleschi did not reveal the method with which he intended to carry out the construction: after so many sacrifices and so many arguments, he did not want others, particularly Ghiberti, to take possession of his ideas.

Only Brunelleschi would have been capable of managing the construction site of the Dome. The section of the dome between the first and second passageway was built between 1422 and 1427. The first date corresponds to the decision about the revision of the program, dated 13th of March 1422. The problem concerning the weight of the structure arises again two years later, and, to this end, it was decided to reduce the transverse dimensions of the middle spurs and to anticipate the use of brick masonry. Moreover, the increase of inclination of the resting beds of the blocks of stone and the necessity of converging on the axis of the Dome make the measure of 24 *braccia* of the stone structure inapplicable. Thus the Dome develops up to the level of the architraves of the passageways of the spurs, above which begins the oblique adaptation of the laying beds with inclination of approximately ten degrees. All this allowed Brunelleschi to take his revenge on Lorenzo Ghiberti. At the end of 1423, he pretended to be seriously ill; he was so good at acting that people started believing his death was really close at hand. Responsibility for the construction fell on Lorenzo, who quickly found he was incapable of continuing. The construction work stopped. At this point, by a miracle, Brunelleschi recovered and started a campaign of denigration against Lorenzo.

The Opera del Duomo understood that only Brunelleschi could continue the project and raised his salary to one hundred florins per year (an enormous sum for that time). Ghiberti was suspended from his role as director from July 1425 to February 1426, when he was reappointed with a monthly salary of three florins, and soon after, it was taken away from him: that meant there was no longer any need for him. Brunelleschi's salary was then raised to one hundred florins per year, with daily presence required at the construction site.

The construction site was very safe although placed at a significant height; in fact, Brunelleschi had foreseen even its slightest details, particularly for the workers' safety. It was reported that only one death occurred during the entire duration of the construction (which lasted 16 years). In order to reinforce the base of the Dome, the construction was initially made of stone; and afterwards of brick, in order to make it lighter. For this reason, Brunelleschi designed two domes: one internal and one external. Had only one Dome been built, it would have been four and one-half meters thick. It would have been too heavy and, most likely, the underlying structures would not have supported such a heavy weight. On the contrary, the internal dome, which is the supporting one, is 2.40 m thick at its base and 2.20 m thick at the top. The external one has the job of protecting the internal dome from bad weather and has a thickness variable from 90 cm at the base to 45 cm on the top. Moreover, with the construction of two domes, it was possible to obtain an internal passage that allowed access to the lantern's platform. On the subject of measurements, an interesting fact should be known. When observing the Dome, one can immediately notice the feeling of a harmonious balance between its parts. Actually, this balance exists and is not unintended. The base of the dome is located 55.00 m above the ground, the drum measures 13.00 m, the lantern is 21.00 m high, the height of the Dome is on average 34.00 m (32.00 m for the internal dome and 36.00 m for the external one). Thus we obtain the following series of numbers: 13, 21, 34, and 55. These are precisely the four consecutive numbers of the Fibonacci series; they are very significant numbers and are found in several aspects of nature and the arts.

They have remarkable properties: one is that the relationship between one Fibonacci number and the next approaches the "golden ratio," which is the most harmonious found in art and nature. For this reason it was frequently used by the ancient Greeks, who clearly understood harmony. This ratio already belonged to the parts of the Cathedral beneath the Dome. Mathematics is very helpful in determining the harmony of things. One can only speculate about Brunelleschi's extensive knowledge of plans and building techniques of the Eastern mausoleums, probably derived from stories told by those who had seen the buildings first-hand; for example, the double dome built without centering in the mausoleum of Soltaneih between 1304 and 1312.

At a height of about 4.60 m *(eight 'braccia')* above the treading plane, is the only wooden chain, made of 24 beams, and surrounding the entire Dome. This chain is internal to the first sector's inter-space and anchored to the spurs. This ring is needed to "narrow" the Dome at its base, in order to oppose the forces that move outwards. The construction of this ring was a bit of an engineering feat; to accomplish it, Brunelleschi had to know mechanics and materials.

The beams forming this ring are connected with iron brackets and studs, and its execution surely must have been arduous. It was precisely this problem and its execution that gave Brunelleschi the chance to show his cleverness, pretending to be ill and leaving this task to Lorenzo Ghiberti. Moving upwards, one can notice the different placements of the bricks, which constitute the rows that form the corner ribs, that is, those connecting two adjacent *'vele'*. Besides the corner ribs, there are also the middle ribs: two for each '*vela*', serving to support the structure. Altogether, there are 8+16 ribs that wrap around the entire construction.

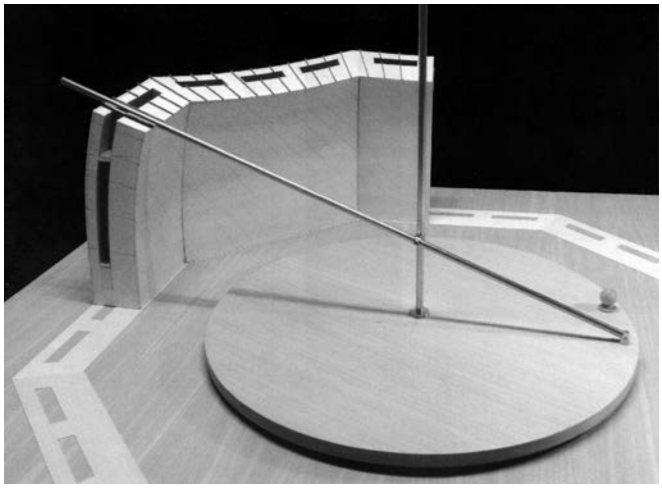


Fig. 3. Model made by architect Franco Gizdulich that demonstrates that the laying planes of the bricks are the same for adjacent 'vele'.

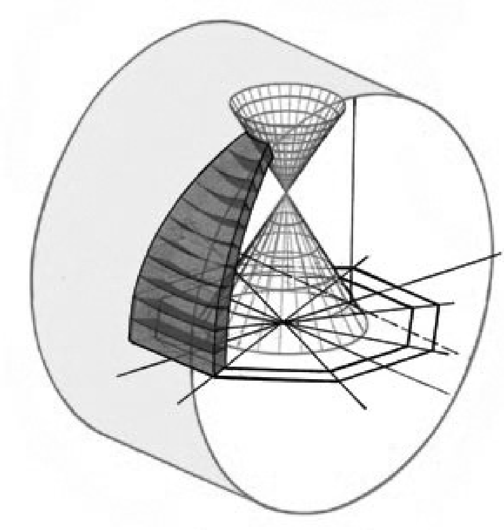


Fig. 4. Axonometric scheme of the course of the' corda blanda'.

The rows of bricks that make them up are steeply inclined, and their inclination increases to about 60°. The corner ribs and the middle ribs are connected by a series of arches (nine for each *'vela'*). These arches are used to link the external *'vele'* with the internal ones; they "dispose" the weight of the external '*vela*' onto the corresponding internal *'vela.'* Walking through the corridor, one notes the bricks that form the *'vele*;' we find the internal *'vela'* on the right side and the external one on the left side. The bricks have two different placements. Some bricks are placed with their longer side perpendicular to the horizontal plane; as mentioned: this configuration is called “herring bone.” The “herring bone” pattern of bricks in the Dome has the same function as in circular-based domes: it prevents the sliding of bricks during construction. In fact, given the critical value of the laying beds (‘*letti di posa*’) of bricks towards the Dome’s axis (approximately at the level of the treading plane), the designer makes use of this specific configuration of the bricks. We notice that the “herring bone” technique is not found in the Baptistery. The “herring bone” bricks depict a curvature similar to that of a spiral

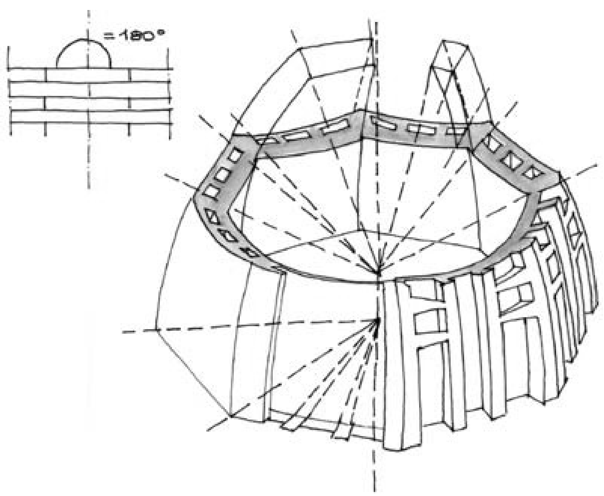


Fig. 5. Spatial visualization of the curved line, called ‘corda blanda’.

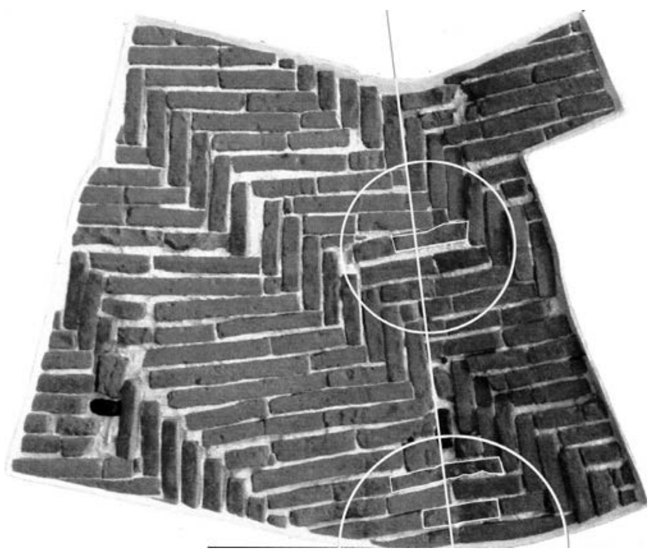


Fig. 6. Models in fiberglass insistent on portions of the ‘vele’ 6 and 7. Corner bricks are clearly visible, have an unclear shape, and were made on the spot by chiseling a whole brick.

staircase: such a curve is called a “cylindrical propeller.” After the second passageway Brunelleschi employs the “herring bone” device. The staircase that leads to the top of the Dome also is a spiral, but curves in the opposite direction to that of the “herring bone.” This is not accidental, because in this way the structure is strengthened. From July 1433, the construction continues from the level of the third passageway, placed at a height of 23.00 m or more, up to the concluding section, begun in 1435. At the level of the third passageway, the inclination of the laying beds of the bricks reaches approximately 40° with respect to the horizontal plane, with clear problems caused by the tendency of the wall layers to slide. As mentioned, Brunelleschi brilliantly solved this problem with the “herring bone” technique. Here, this method is more frequently applied: the distance between two adjacent “fish-bones” reaches 75 cm. For a quick setting of the bricks, lime putty or mortar made of finely cut sand, are used. The fish-bone wall structure of the ancient Selgiuchidi (10th century) or the later mosques of Isfahan and Ardistan (11th century) are quite close to Brunelleschi’s structural language and technique. Other bricks follow a curved line, called *‘corda blanda,’* that moves towards the ground and goes back up after the halfway mark of the *‘vela*.’

The use of this technique derives from the fact that Brunelleschi applied a technique typically used in circular-based domes to the Dome of Santa Maria del Fiore, which has an octagonal base. In this type of dome, the bricks are placed along the circumference: the ‘parallels,’ so-called because they are located on planes horizontal to the ground level. In addition, the parallels perpendicularly intersect the meridian lines. In spherical domes, bricks located on the parallels are inclined towards the center; thus, they lie on conical surfaces that have their vertex in the sphere’s center. Because Santa Maria del Fiore’s Dome has an octagonal base, if the bricks had been placed following octagonal rings, they would have formed corners at the joining points of two adjacent *‘vele*.’ Discontinuities of structure would have appeared exactly where the main concentrations of stress occur. This fact would have caused damage dangerous to the Dome’s stability. Brunelleschi had noticed that the octagonal dome of the Cathedral was built in exactly that way, and that damage had appeared exactly at the corner points. But in this case, there are no stability concerns, since the external construction supports the structure of the dome. The Dome of Santa Maria del Fiore did not have any external structures that sustained it. So, in order to avoid the laying plans of the bricks forming angles at the corners, Brunelleschi built it as if it were a circular-based dome. To this end, at the corners, bricks belonging to the adjacent *'vele'* on the same level were placed. Each of these levels is perpendicular to the corresponding corner rib. Continuing with this placement of the bricks in the *'vele'* as well, they are arranged following curved lines, so that they are perpendicular to the Dome's meridians, not only at the corner ribs points, but also along the *'vele.'* Placing the bricks according to curves that belong to cylindrical surfaces (the *'vele'*), an instance of perpendicularity occurs between the laying plans of the bricks and the meridians, exactly as occurs in circular-based domes. The only difference from the latter is that the parallels of spherical domes, which are perpendicular to the meridians, are effectively parallel to the ground, whereas the Dome's parallels (that is, the '*corde blande*'), which are located on a cylindrical ellipse, are always perpendicular to its meridians, but are not found on planes perpendicular to the ground. One notes that the Dome's meridians are not circumferential arches, as in the spherical ones, but elliptical arches. Continuing towards the top of the Dome, we find ourselves in the third passageway, where significant inclination of the bricks can be noticed. In order to reach the Dome's summit, one must climb a staircase located above the extrados of the external *'vela*.' Beneath it is a layer of bricks 2.20 m thick, and then a drop of almost 90.00 m. A competition, won by Brunelleschi, was also announced for the Lantern, of which he made a wooden model, still extant at the Museum of the Opera del Duomo. Its construction started in 1446, the year of Brunelleschi's death. His stepson, Buggiano, managed the construction work. Luckily, he followed Brunelleschi's plan to the letter, because the Lantern has a very specific function in relation to the statics of the Dome. The corner ribs converge towards a ring with a diameter of about 6 meters. Even if Brunelleschi had tried to give the Dome the best possible shape, the forces that act on the Dome are such that the corner bricks themselves tend to curve outwards, on their own sides. And it would have been a disaster: the Dome could have been at risk of opening up and collapsing. The Lantern has the function of counteracting these outward forces, wedging the structure in: it fits into the Dome with a wedge, which is the '*serraglio*.' Every element of the Dome has a very specific function. Thus, the heavy weight of the Lantern (it weighs more than 750 tons), which places a burden on the angular '*setti*,' aligned with the axis of the spurs of the Dome, does not weaken it, but instead, makes it more stable. A ring-like path, placed at the height of 33.00 m, crosses the angular '*setti*.' For a height of 1.30 m above the treading plane, one notes significant inclination, approximately 60°, reached by the last layers of the bricks of the '*vele*.' Exiting the Dome, we are at the base of the Lantern.

2. First investigations on the Dome[1]

There are several theories and hypotheses about the static functionality and structural solutions of the Dome of Santa Maria del Fiore, but there is a lack of experimental data and[1] instrumental investigations. The dome structure still remains unaffected and intact and the possibility of surveying the internal portions is very limited. Thus far, the building has been thoroughly studied only in those parts of the structure that are directly visible, while a margin of uncertainty still remains about the internal areas, which can be examined only with non-intrusive techniques, due to its artistic value. In the last decade, with the introduction of the most recent technologies in the surveying and analytical fields, several types of remarkable surveys have been carried out and have revealed very interesting data.

3. Survey with laser scanner

In February 2004 the Florentine society of General Engineering carried out the first survey done with a laser scanner by the Galileo Siscam Technology in collaboration with Codevintec of Milan. For the survey, an instrument of the high speed type was used, considered to be the most adequate for its capacity as well as speed of acquiring data. This instrument allowed for defining of the volumetrics of the dome globally and for documenting of all surfaces in a continuous manner with a

1. See R. Corazzi, G. Conti, *Il segreto della Cupola del Brunelleschi a Firenze - The Secret of Brunelleschi's Dome in Florence*, Pontecorboli Ed., Firenze 2011.

cloud of points that covers the artifact evenly. The instrument used for the survey had these characteristics:
- accuracy: 3 mm;
- dimension of the spot: 29 mm at 100 m;
- speed of acquisition: 2000 points/sec.

Observed were three *'vele'* in the northwest part of the drum of the cathedral, the external extrados, and a part of two *'vele'* (northwest part of the first landing).
A later laser scanner survey was carried out in 2006 by the firm Geoarte S.T.A. (Sistemi Tecnologici Avanzati) s.r.l. of Castelfranco di Sopra (Arezzo).
A Leica HDS 3000 «high-speed» instrument was used, and had the following characteristics:
- accuracy: 6 mm at 50 m;
- dimension of the spot: 6 mm at 50 m;
- acquisition speed: 1800 points/sec;
- camera with resolution of 1 Megapixel (1024x1024).

Here, the whole soffit of the internal dome was surveyed. This covering of points was the object of this study intended to research the geometry of the dome's outline, which was then compared with that obtained by the FO.A.R.T. firm's photogrammetric survey.

By examining the survey done by the company FO.A.R.T. with a laser scanner, it was possible to define, for the first time using modern technologies, the geometry of the internal cap. By using software for the management of the data displayed by the laser, planar and sectional profiles of the structure were defined and then compared with regular geometrical figures (octagon, circumference and ellipse). The aim of employing this modern survey method was to acquire detailed geometrical information and to verify the conic that best approximates the ribs and the median sections of the '*vele*.'

With the survey conducted by Geoarte, a "cloud" of points, relative to the totality of the internal Dome, was obtained. The cloud was re-evaluated with software, Cyclone 5.1 by Leica, that allows us to acquire data from the scanner visualized both through the different colors of the instrument with a result similar to a photograph, as well as a multi-hue intensity map that facilitates the selection of the cloud's points.

The above geometrical profiles of the soffit of the internal dome were acquired defining them in horizontal and vertical sections.

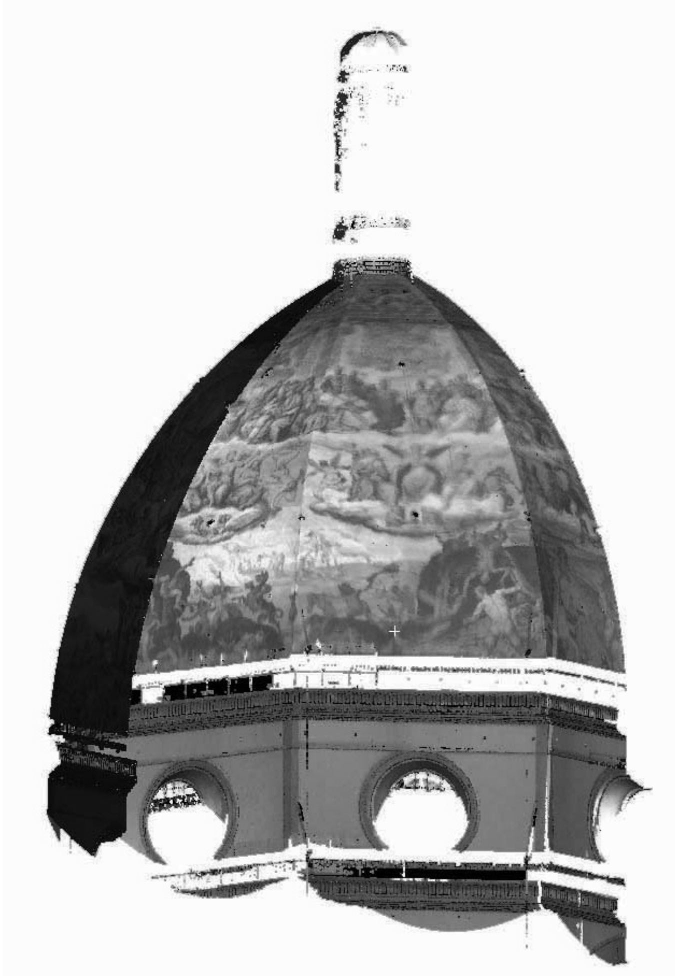


Fig. 7. Visualization of the cloud of points.

In the preliminary stage of the research, surveys done in past years were examined and published by different authors.

Among these works, the focus was on the photogrammetric survey done by the company FO.A.R.T. and published in *"La Cupola di Santa Maria del Fiore. Il rilievo fotogrammetrico,"* edited by Riccardo Dalla Negra. This work was the most recent of all and was used to analyze prior observation on the geometrical profiles of the caps. *"It was observed that the conic that best approximates the external profile of the marble ribs is an ellipse with low eccentricity and with a horizontal semi-axis smaller than the vertical one, while their projection on the median section plane of the 'vele' is well approximated by an arch of circumference. Similar observations, but with a negative sign, can be carried out in the middle sections of the 'vele,' where it seems that the elliptical profile (already* *described) of the soffit of the internal cap is juxtaposed to the circular profile of the extrados of the external cap.*

This matter, if verified, raises an interesting question relative to a possible "optical correction" of the ribs and of the external cap. because "(...) it appears more magnificent and inflated,S or a contributory cause."

For this reason, the profiles obtained with the laser scanner survey were supported by the other survey described and were inserted into that theory.

Three horizontal and eight vertical sections were measured by Cyclone and obtained by having a different crossing plane intersect the clouds of points each time, and deducing portions with infinitesimal width, which describe the geometry of the internal dome. The portions were then examined by software, Rhinoceros Version 3, in order to re-evaluate through projections onto a plane, thus obtaining a poly-line able to define with precision the profile of the intrados of the internal dome.

1 Inner profile of the cloud of points of the mid-section of the two *'vele'* (Geoarte survey).
2 Theoretical circonference.
3 Theoretical ellipse.
4 Theoretical axes of ellipses.
5 Ellipse center.
6 Inner profile of the cloud of points of the mid-section of the two ribs (Geoarte survey).

The horizontal sections cross respectively at +4.00 +13.00 and +24.40 m from the upper balcony. These altitudes were dictated by the necessity of carrying out the sections on the same points as in those reported by the survey FO.A.R.T., in order to use them as starting data into which the profiles, obtained through the laser scanner, could be inserted. In fact, a profile of the octagon of the intrados of the internal dome was obtained and, afterwards, imported into the software ADT 2005 by AUTODESCK in order to insert it into the maps made by FO.A.R.T. with a 1:50 scale.

Subsequently, graphs that were compiled outline the differences between the profiles obtained by the clouds of points of the scanner and those related to a regular polygon, in order to confirm, once again, the imperfection of the octagon of the hollow space of the drum, which presents sides that differ by almost 0.623 m.

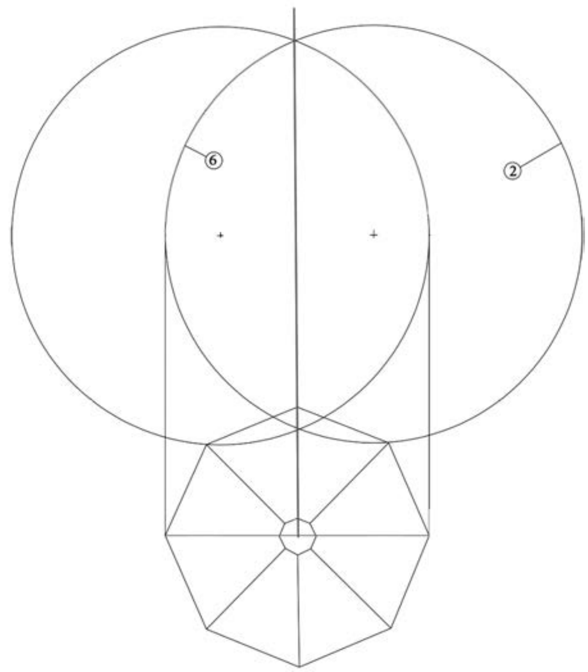


Fig. 8. Plan and section of points at the mid-point of the two ribs including the verification with the theoretical circumference.

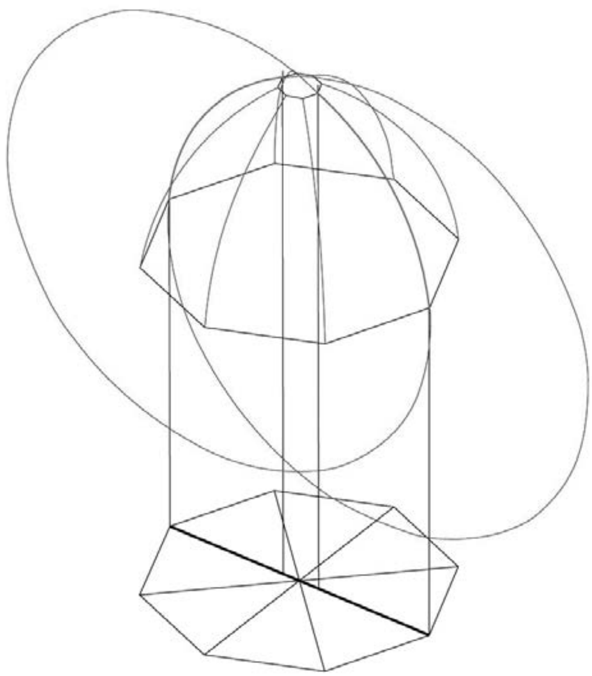


Fig. 9. Spatial visualization of points of the two tibs with an indication of the theoretical circumference.

Moreover some enlargements were made in the 1:50 scale to define, under a metric viewpoint, the most remarkable points in order to outline the differences mentioned above. In particular, the minimum and maximum differences measured on each plan can be summarized as follows:

Differences on survey between Geoarte and profile of the regular octagon (cm)
Minimum Values

Map at Height +4.00 m	83.47	117.46
Map at Height +13.00 m	71.47	103.76
Map at Height +24.40 m	13.30	68.82

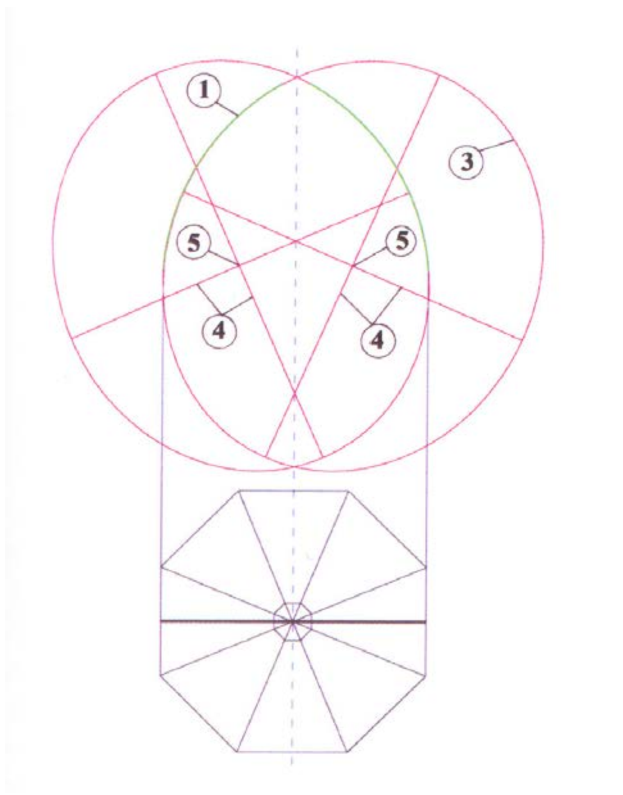


Fig. 10. Plan and section of points at the mid-point of the two ribs including the verification with the theoretical ellipse.

Plan laser scanner and the possible approximation with an arch of circumference for the corner ribs and with an ellipse for the middle of the '*vele.*'

Once again, some enlargements to a 1:50 scale were made to metrically define the most remarkable points in order to outline the differences mentioned previously.

Concerning the comparison with the arch of circumference, the differences are minimized, so that the two geometrical profiles can be considered exactly overlapping and, therefore, the sections of the ribs can be assimilated to an arch of circumference. Moreover, it is confirmed that the pointed sixth vault of the arch of circumference can also be comparable to the profile of the ribs, with a radius of approximately 36.00 m (36.80 m, 36.10 m, 36.18 m, respectively for the section 1-5, 4-8, 3-7, 2-6).
With respect the comparison of the profiles with those of an arch of circumference, there is an almost perfect correspondence, which allows assimilation of the section of the mid-portion of the '*vele*' with the geometrical figure. The ellipse that best describes the profiles of the middle sections has the same dimensions for all the sections, and, therefore, for all the '*vele:*' the minor axis measures 40.63 m, and the major axis 56.67 m.

The ellipse that best approximates the profiles of the caps presents an axis inclined at 20⁰, in respect to the median plane of the upper balcony. This inclination was researched by considering the studies conducted in that field by Lando Bartoli, and published in *"La rete magica di Filippo Brunelleschi. Le seste, il braccio, le misure,"* pp. 59 and 60. Geometrical verification of comparability

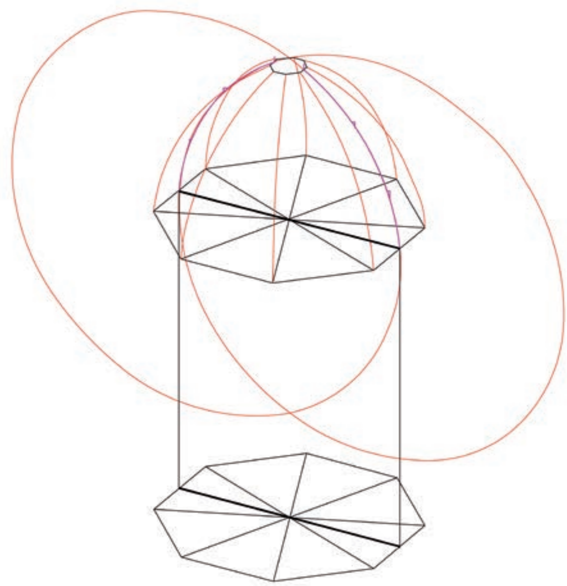


Fig. 11. Spatial visualization of the points midway portion of the two 'vele' with an indication of the theoretical ellipse.

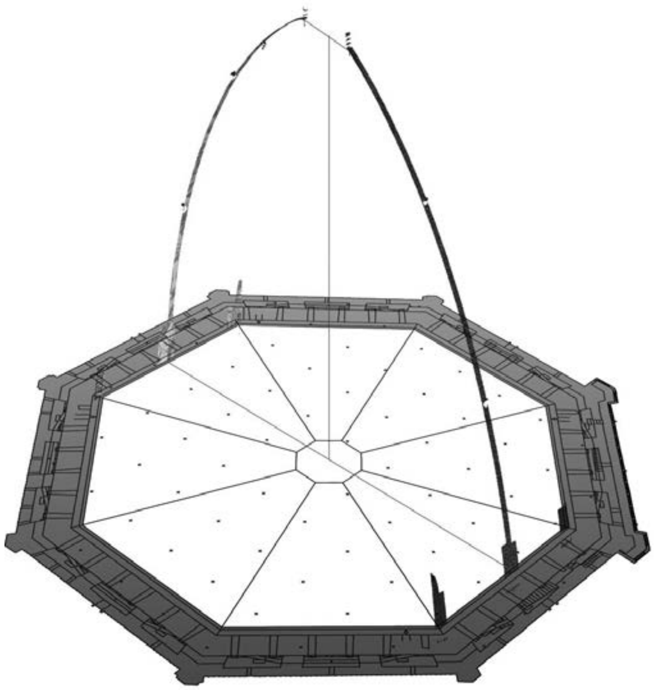


Fig. 12. Plan and section of the cloud of points of the mid-section of the two 'vele'.

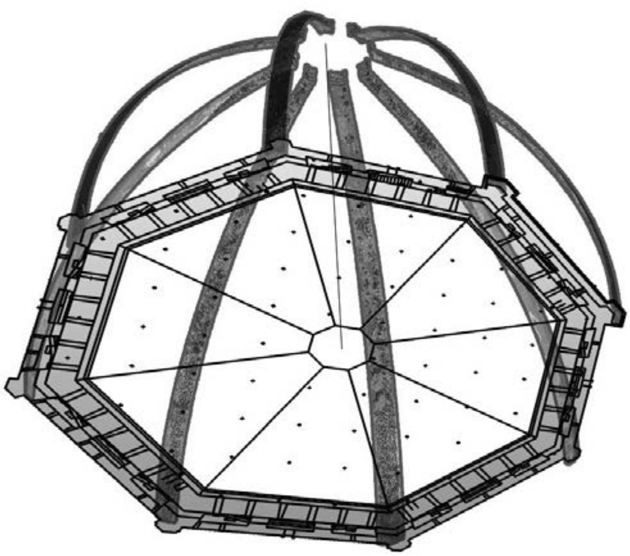


Fig. 13. Spatial visualization of the clouds at the points of the ribs.

of the profiles in the mid-sections and the ribs were also done on two *'vele'* of the external cap.

Because the laser scanner survey, mentioned above, focused only on the internal cap, the photogrammetric survey, made by the company FO.A.R.T., was used for these verifications; therefore, it was possible to confirm, once again, the comparability of the profile of the middle sections with an ellipse and the profile of the ribs with an arch of circumference, and not the opposite, as stated by Riccardo Dalla Negra, whose considerations are therefore incorrect. *(From the photogrammetric survey - the Dome of Santa Maria del Fiore - edited by Riccardo Della Negra, Sillabe, Livorno 2004, p.37 (...).*

"More complex is the matter of the profiles of the extrados, where it is more evident both that the external marble ribs do not follow the course of the internal ones, and that in the sections of the mid- portion, the cap presents some depressions.

It was possible to observe that the conic that best approximates the profile of the actual marble ribs is an ellipse with low eccentricity and with a horizontal semi-axis greater than the vertical one, while their projection on the plane of the middle section of the '*vele*' nicely approximates an arch of circumference.

A similar observation, although with an opposite sign, can be carried out in the middle section of the '*vele,*' where the circular profile of the extrados of the external cap seems to overlap with the elliptical profile of the intrados of the internal cap, already described above. This matter, if verified, raises an interesting question relative to a possible "optical correction" of the ribs and of the external cap. because "(...) it appears more magnificent and inflated, or a contributory cause...)."

In relation to the arch of circumference comparable to the profile of the rib, it is confirmed that it is a vault of pointed fourth, with a radius equal to 43.00.

The ellipse comparable to the profile of the middle section has its minor axis equal to 51.27, is inclined at 32° to the middle plane of the upper balcony, and has its major axis equal to 68.36 m. To evaluate the results obtained, the geometrical verification of such profiles was linked to a mathematical verification.

A curve was determined for each profile; this curve best approximates the points mentioned following the "*method of the minimum squares"* or "*best fit,"* which consists in establishing the curve to be determined *'a priori.'*

1. Poitend fifth arch.
2. Poitend fourth arch.
3. Circumference coinciding with the bottom surface of section of Dome.
4. Circumference coinciding with the extrados of section of Dome.
5. Floor of the supper balcony walkway at the base of the Dome.
6. Top balustrade.
7. Inner arch with of mid-section of *'vela'* (Geoarte survey).
8. Theoretical ellipse.
9. Axes of theoretical ellipses (major axis = 56.67 m; 40.63 = minor axis)
10. Ellipse center.
11. Inner profile of the center-line of the *'vela'*.

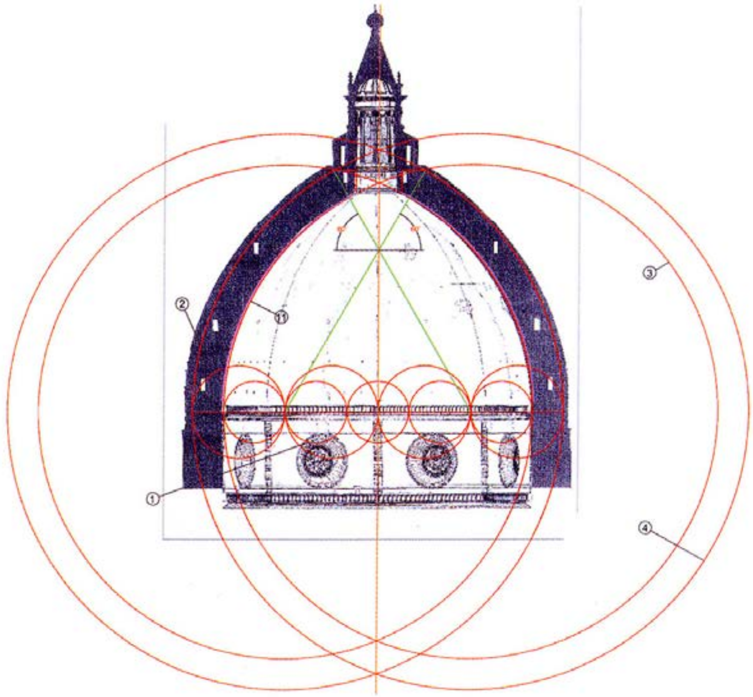


Fig. 14. The section is carried out of the level of the mid point of the 'vele' and is represented by a circumference.

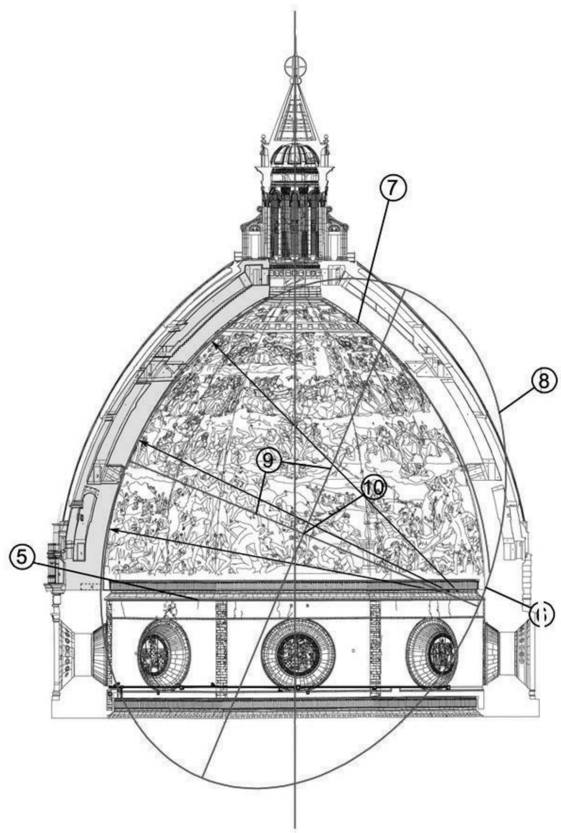


Fig. 15. The section is carried out of the level of the mid point of the 'vele' and is represented by ellipse.

4. Georadar

In May 2002, a first non-invasive geophysical survey with geo-radar was carried out in the extrados of the internal southeast '*vela*'. The aim was to locate and to map the possible presence of cavities or supporting elements found in the masonry's structure. 600 and 1500 MHz antennas were used; they passed through the surface of the '*vela*', perpendicular to the reference plane of the balconies, in an ascending direction and at a pace of 50 cm.

The images recorded by the geo-radar represent the vertical sections relative to each of the linear profiles covered by the mobile antenna on the surveyed surface. For each sector, the measured sections were compared with the different frequencies and, in relation to the surveyed parts, the electrical characteristics, the diffusion speed of the electromagnetic waves, the relative electrical permittivity (dielectric constant) and the depths reached, were examined.

By elaborating the data, it emerged that the section of the Dome is basically formed of two "faces", inside which a "filling" is found. Certainly, the material of the face of the extrados is composed of a texture of bricks, characterized by the rows of the "*corda blanda*" and by the herringbone for a layer not exceeding 70 cm; for the opposite screen, well outlined according to the linear reflections of the signal and registered at 1.75 m from the surface, we hypothesized the same textural characteristics and the same dimensions. For the intermediate section, defined as "filling", it is possible to hypothesize a sack of variegated material, or a different layout of the clay elements. Starting from the upper space, evidence of possible connecting elements among the parts was emphasized. They are fairly linear projections, which commence from the intrados of the cap; are equally inclined and seem "drowned" inside the layer of the "filling".

In February 2004, a second non-invasive survey was completed with a geo-radar method, as part of the project Optocantieri (PRAI - Region of Tuscany) and conducted by a Florentine company General Enigineering - Galileo Siscam Technology. This survey was intended as a first attempt to execute a gauging of the instrumentation and to find the best configuration to penetrate the entire depth of the dome with electromagnetic impulses. 500, 900 and 1500 MHz antennas were also used for this experiment. The individual characteristics and potentialities of each antenna were speculated upon and, finally, the decision was made to use the 900 MHz antenna, set at a time of 40 ns, with an average dielectric constant for the observed object equal to 5. This antenna best covered the entire layer of the masonry, penetrating it at its maximum frequency and, thus, with the best resolution; the time was set at 40 ns because it allows for investigation of the entire layer with a contained margin. The same portions of the masonry, previously surveyed, were examined with a geo-radar, in order to allow significant comparisons. It was possible to survey up to depths of 5.00 m from the treading plane of the balcony, by tracing, with chalk, a reticule of orthogonal reference with a pace of 50 cm; in total, 174 profiles, both vertical and horizontal, were completed, and each profile was surveyed with antennas with different frequencies and with variable configurations. In detail, the following combinations were tested:

- 500 MHz antenna set both at 40 ns (corresponding to approximately 2.7 m of depth) and at 70 ns (4.7 m of depth), and with a coefficient (dielectric constant) equal to 5;
- 900 MHz antenna set at 40 ns (corresponding to approximately 2.70 m of depth), with a coefficient (dielectric constant) equal to 5 and with a surveying pace of 50 cm;
- 1500 MHz antenna set at 20 ns (corresponding to approximately 1.35 m of depth), with a coefficient (dielectric constant) equal to 5 and with a surveying pace of 50 cm.

It is clear that the traversed medium has a different quality; the first stretch of the section of the Dome, from 0 to 65 cm, is characterized by many micro-reflections, typical of a stone structure, where the joints of mortar generate interferences diffused by the signal, because there are many transitions from stone to mortar; the second layer of masonry, on the contrary, from 65 to 200 cm, results homogeneous and lacking in small reflections diffused by the previous layer. It seems to be a more compact and homogeneous masonry, lacking in gaps.

In May 2007, a further non-invasive survey with the geo-radar method was carried out by the company "IGeA s.a.s." in Borgo San Lorenzo (Florence). The study concerned one portion of the extrados of the east

1 Filling

2 Brick herringbone

3 Floor bricks

4 Transmitter. Transducer.

5 Electromagnetic pulse response of a material

6 Electromagnetic pulse sent from the radar

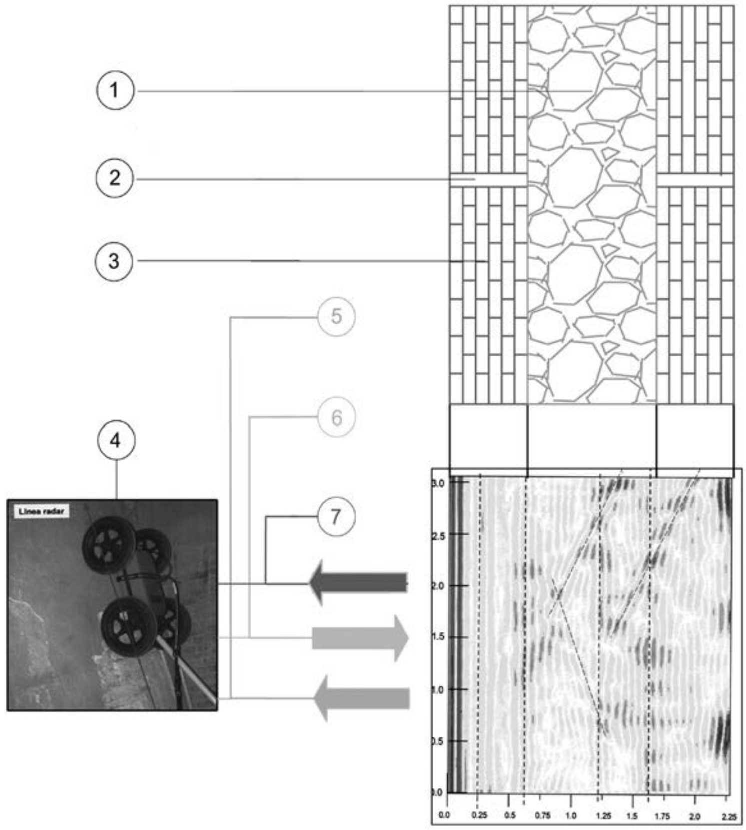


Fig. 16. Overlapped radargram executed in 2002 and drawing of the masonry structure.

vela of the internal Dome; in particular, an area 3.30 m. wide and 2.00 m. high was examined by subdividing it into 48 vertices distributed on a regular reticule with a pace of 40 cm. Two antennas were used: one has a frequency of 500 MHz and the other 1500 MHz, in order to investigate the surface both in depth (500 MHz) and with a high resolution (1500 MHz). In fact, in a relation with inverted proportionality, the increase in frequency corresponds to a diminution of the depth of the possible survey; however, in a relation with direct proportionality, the increase in frequency corresponds to a rising of the resolution of the survey, that is the capacity to detect elements with gradually smaller dimensions. The aim of the survey was, once again, to detect and map the eventual presence of holes or reinforcing elements located in the layer of the masonry; to extend this study, by adding information to that obtained in previous years, the survey was carried out in the vela adjacent to that examined in 2002 and 2004. The surveys were executed along the vertical and horizontal lines of the reticule, specially drawn with chalk on the surface of the vela. The profiles were placed so that they would cover the surface of the vault with alignments orthogonal to each other, according to the reticule: longitudinal profiles 3.20 m long and transversal profiles 2 m long, placed at a distance of 40 cm. The survey phase consisted in a series of measurements by simple reflection, with receiving and transmitting 500 MHz antennas, to estimate the speed values of the electromagnetic waves. The measurements by simple reflection were acquired at a continuos rate and with density of approximately 100 traces per meter. The survey was conducted with radar instrumentation Zond 12 by Radar System Inc. and Zond antennas with frequencies of 500 MHz and 1500 MHz.

In total, 8 vertical and 6 horizontal profiles were done, each of them measured by the two antennas with different frequencies. In detail, the following combinations were attempted:

- 500 MHz antenna, set at 40 ns for its temporal depth

of exploration and with a dielectric constant equal to 4 (about 3 m of surveying depth);
- 1500 MHz antenna, set at 40 ns for its temporal depth of exploration and with a dielectric constant equal to 4 (about 3 m of surveying depth).

In particular, it was believed that:
- with the 500 MHz antenna, surveying depths were reached adequate to examine the entire layer of the dome with a detail of about 35 cm;
- with the 1.5 MHz antenna, surveying depths were reached adequate to examine the entire layer of the dome with a detail of about 10 cm.

The responses of the radar, obtained for the single sections, were then reproduced in a dimensional and tri-dimensional shape, with representations consisting of iso-surfaces of the value of the reflecting amplitude that allow the outlining of the volumes characterized by a georadar response value inferior or superior to a set threshold value. In particular:
- at a depth of about 7 ns, corresponding to approximately 50 cm from the surface of the extrados of the Dome, a continuous reflection, which is also parallel to the surface, is found. The material inside this first layer produces small reflections and diffused disturbances;
- at a depth of about 16 ns, corresponding to approximately 1.30 m from the surface of the extrados of the Dome, a continuous reflection, which is also parallel to the surface, is found and turns out to be weaker than the previous one (evidently due to a lower signal energy). The material found in this portion of masonry is more homogeneous than the previous one and does not show further micro-reflections and/or disturbances;
- at a depth of about 28 ns, corresponding to approximately 2.20 m from the surface of the extrados of the Dome, a strong continuous reflection is found, a characteristic attributed to the extremity of the masonry and, thus, of the intrados of the vela. The material found in this portion of the masonry seems to have the same qualities as that surveyed in the area of the extrados, even though the energy of the signal (at this point fairly reduced due to depth) does not allow an equally precise interpretation. Moreover, continuous reflections are found and present an inclination of 35°-40° with respect to the surface of the extrados of the dome. Such reflections are not always very evident, and are overlapped in some sections, while in others, they are almost invisible. The geometry is also visible in the parallel and longitudinal sections.

The three different surveys, executed with georadar, were compared in order to get detailed information, and are also to be evaluated in relation to the data of further geophysical prospections (metal detector and electric tomography) that have been carried out in the vela by the company “IGeA s.a.s.” from Borgo San Lorenzo in Florence**.**

5. Tomography

To execute the tomographic survey on the Dome, geometry was used; it foresees the position of a “remote pole”, that is, placed at a theoretically infinite distance. In our case, considering the relatively small dimensions of the reticule, we placed the remote pole at the top of the dome, at a distance of approximately 40.00 m. The results outlined the presence of materials with values of resistivity fairly different from each other, In particular,a first , superficial level (extrados side) was identified, having a thickness of about 0.50-0.60 m. and with values of resistivity between 4.5 x 10^4 Ωm and 5.5 x 10^4 Ωm. A second level of more uniform and conductive materials was found, in stratigraphic discontinuity with the parallel plane, with a thickness of about 0.80 meters and with values of resistivity between 1.5 x10^4 Ωm and 2.5 x 10^4 Ωm. We can presume the existence of a third layer, up to the extrados, of materials with a larger resistivity, probably assimilable to the extrados area. Due to the limited extension of the rectangular reticule employed , the depth of the exploration was limited, therefore the measures relative to the intrados area are necessarily smaller with less details. The central area of the dome (that with the lowest values of resistivity) has values comparable to those in the section where a sandstone buttress is located, in correspondence with the low-central part of the tomographic grid. The above can assume an analogy between similar lithotypes, at least from a mineralogical viewpoint. The conclusive joint results of the various methods utilized on the surveyed vela do not allow for the possible presence of ferrous material, possibly used as a structural framework of the Dome of the Duomo. The Ferrous material found is related to connecting belts between stone or masonry elements, but with dimensions not larger than several inches. In the passageways, by contrast, the metallic and/or ferrous belts have larger dimensions and lengths, often larger than one to two meters and, above all, show a recurring geometry in the examined spaces. The structure of the Dome can be likened to a structure made of three layers of quite uniform thickness (at least in the surveyed vela) but made of lithotypes differing both for their electromagnetic reflectivity and for their electric resistivity.

Structural anomalies are found and are due to the operative and building modalities of the Dome ('*buche pontaie*') putlog hole; also "structures" or "planes" are found in the central section of the vela and having transversal and inclined geometries, for which an attentive geometrical-structural analysis has to be executed in order to understand their meaning. To complete this study, radar surveys could be carried out in the intrados side of the Dome, to verify, at least from an electromagnetic viewpoint, the real presence of the third layer and its characteristics in the situation of a maximum electromagnetic-induced signal, rather than compromising its survey with the dispersion caused by crossing the two previous layers.

6. Endoscopy

In order to investigate and unfold the secrets of the dome, and also to confirm and compare the results obtained from the surveys, a study, conducted with the endoscopic method also used in the medical field, was done to reveal the types of materials that form the Dome.

This type of survey is non-invasive, because the research is carried out in the existing apertures and fractures of the '*vele*'. Moreover, artificial holes were used and, as we will discuss later, were located on four of the eight '*vele*'. The use of this instrumentation was possible through the presence of apertures and fractures, inside which either a rigid or flexible probe of the endoscope was inserted.

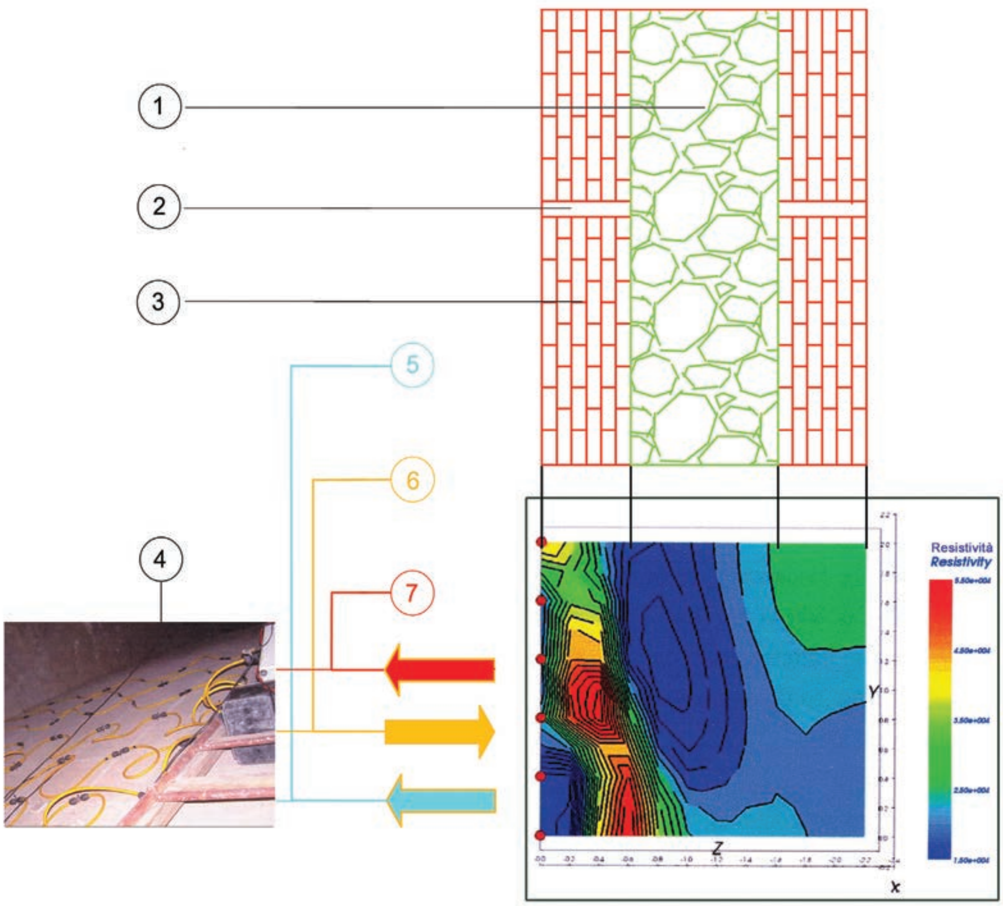


Fig. 17. Overlapped radargram executed in 2002 and drawing of the masonry structure.

Through the optical system, the structure of the masonry of the Dome was observed, photographed and filmed at different depths. Several shots were taken and the results and various reports will be discussed. Some images of the endoscopic survey on the same vela where other studies were also done, will be presented. It is important to note that, during the accurate survey of the surface of this vela as shown in the picture, a core boring was discovered (Ø cm 8 and 2.00 m deep), which was previously discovered by other researchers and which gives important information about the type and characteristics of the masonry structure. With careful research, it emerged that four coring procedures, the images and functions of which are listed, were done on four '*vele*'. Moreover, in one of these, an endoscope jacket was used, inside which, as the images show, a certain quantity of coring material was deposited. The carrots no longer exist because they were used for laboratory tests; but their images, provided by the report published by the Department of Science of Construction and by Prof. Berta Leggeri, offered the opportunity, together with the photographs of the internal surfaces of every carrotage, to confirm what was surveyed by the geo-radar and tomographic surveys.[2]

7. Studies carried out on the model

In order to verify the exact composition of the internal bearing dome and to determine the effective composition and position of materials, a model was created in the garden area of the office of the Architecture Faculty in Piazza Ghiberti, Florence. The model was made up of two external facings 75 cm. in size, built of bricks and mortar, and of an internal area made of mortar, stone, bricks and iron (by using Innocenti pipes inclined at an angle of 45°.

The geo-radar survey done on the model used the same instruments as during the surveys done on the inside of

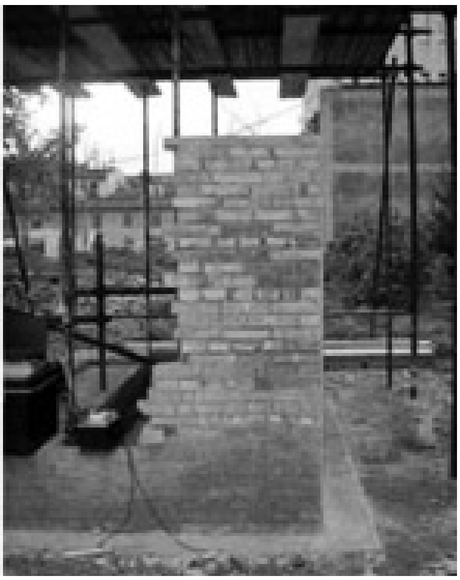


Fig. 18, 19. Construction of the model.

2. Confront S. Di Pasquale, *Brunelleschi: The building of the dome of Santa Maria del Fiore*". p. 125. "However, it should be added that from specific tests of the samples of the masonry layers made with this technique, it was revealed that their collapse has always begun along the mortar joint between the vertical bricks, as has been observed in some sections of the Dome on the occasion of the restoration of the frescoes; also note 52, p. 220. - Finally, because during the initial stages of the experimental research, in agreement with Prof. De Angelis d'Ossat and Cestelli Guidi, we extracted some samples of the materials from the Dome, we were able to carry out some laboratory tests, (L. Barbi, B. Leggeri , V. Vasarri, R. Franchi, F. Fratini, C. Manganelli del Fa, *Experimental Surveys on the materials of the Dome of Santa Maria del Fiore, "Acts by the Department of Construction",* n. 1, 1986) which confirmed a remarkable homogeneity of behavior among the single elements of the "composed material" resulting from the two ingredients, mortar and brick; particularly significant was the fact that initial fractures appeared in the corners of the masonry, in correspondence with the spaces located at the center of the balcony for access to the windows of the internal dome."

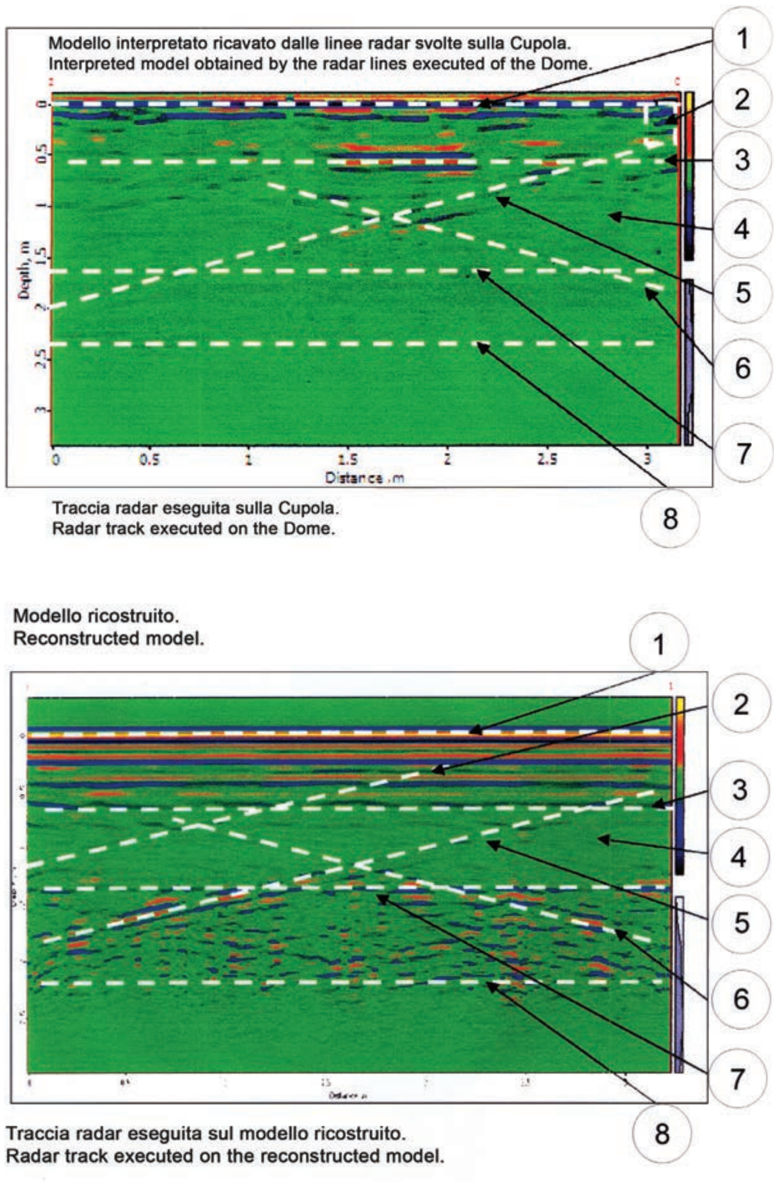


Fig. 20. Comparison of the radar.

1 Work surface.
2 '*Buca pontaia*'
3 First theorized horizontal reflection of the brickwork
4 Theorized area with fill and mortar.
5 Reflection angle suggested as a possible metal framework.
6 Reflection angle suggested as a possible of the brickwork.
7 Second theorized horizontal reflexion at the beginning of the brickwork.
8 Reflection derived from the end of the wall.

1 Work surface.
2 Angle reflection due to metal armor
3 First horizontal reflection derived from the end of the brickwork.
4 Area with fill and mortar.
5 Angle reflection due to metal armor
6 Angle reflection due to metal armor
7 Second horizontal reflection derived from the beginning metal framework.
8 Reflection derived from the end of the wall.

the dome of the Duomo of Florence. The instruments were set using the same parameters of acquisition as in the prior surveys on the dome. The only parameter that was altered is the range of the scale base, slightly lower on the reconstructed model.

The measurements gathered, processed by software specific to the field, allowed the singling out of the same reflection verified in the wall facings of the dome vela of the Duomo, thus confirming that the structure is composed of three layers: the first (extrados) in brick tile 'herringbone'; the second (intermediate) made of a filler mix of mortar and stone elements and/or of tile, while the third (soffit) also in brick tile 'herringbone'. The thicknesses were confirmed through the georadar survey carried out on the model with the permittivity parameter which confirmed the depth of reflection identified in the dome. In correspondence with the metal bars inserted in the model, the same forms of reflection found in the dome were present. The only difference seen was relative to the intensity of reflection noted in the model, slightly greater, probably due to the ferrous section utilized, clearly greater in the reconstructed model. In conclusion, the constructive modalities hypothesized following the georadar, tomographic, endoscopic, and metal detector surveys carried out on the dome, were confirmed by the surveys carried out on the model, concerning both the materials used and metallic reinforcemen.

8. Comparison of the Dome and the Staggia Fotress

The continual reorganization of labor, in the tradition of the Florentine Republic, was usually carried out by the workers of the Opera del Duomo who, precisely in those years, were completing, under Filippo Brunelleschi's guidance, the dome of Santa Maria del Fiore. In fact, on March 15, 1431, in a document found in the archive of the Opera del Duomo, it is reported that the great architect, *General Overseer of the fortifying labor for the Republic*, was sent to Staggia (and to Rencine and to the Castellina) to supervise the fortifi-

Fig. 21. Perspective view of the walled circle of Staggia.

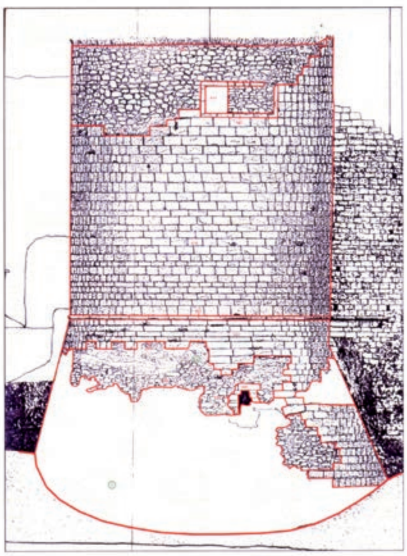


Fig. 22. Main and lateral section stratigraphic perspective.

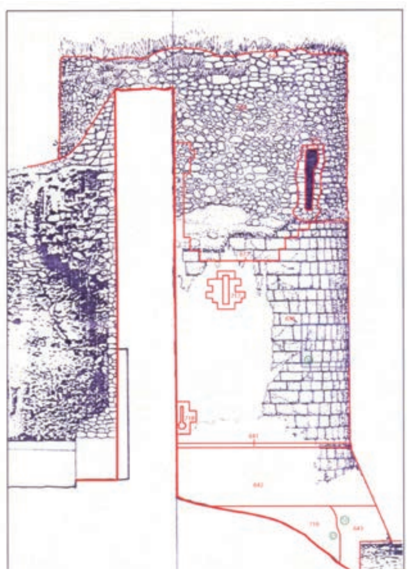
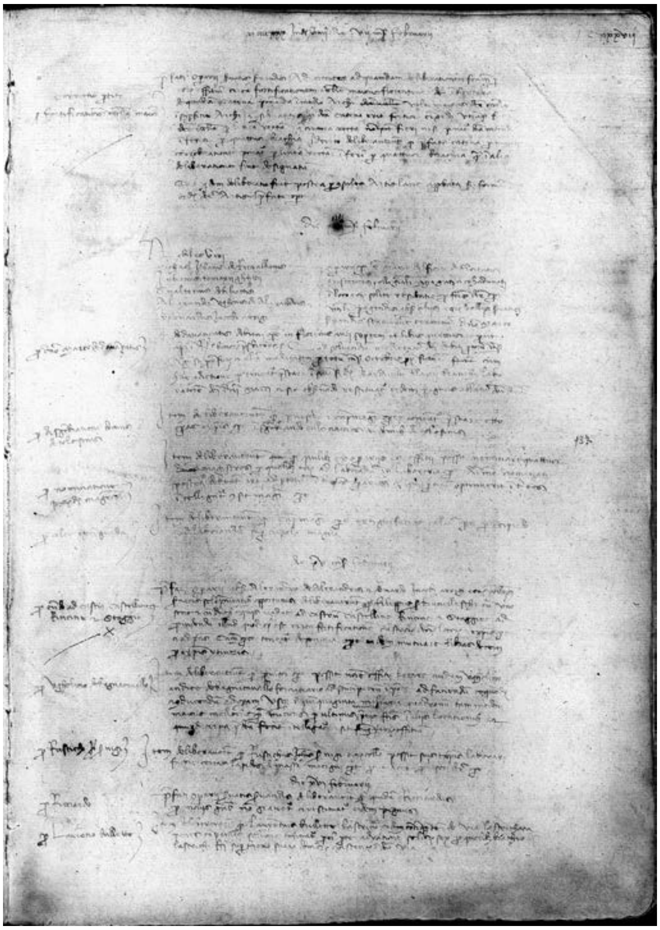
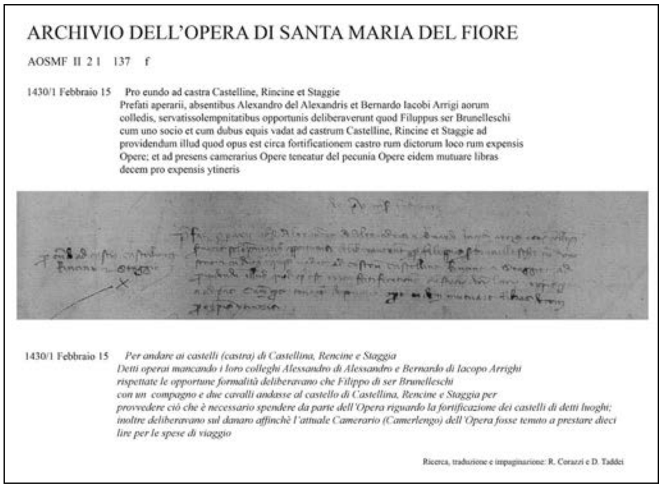


Fig. 23. The intermediate part is quite visible.




ARCHIVIO DELL'OPERA DI SANTA MARIA DEL FIORE

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1430/1 Febbraio 15 Pro eundo ad castra Castelline, Rincine et Staggie
Prefati aperarii, absentibus Alexandro del Alexandris et Bernardo Iacobi Arrigi aorum colledis, servatissolempnitatibus opportunis deliberaverunt quod Filuppus ser Brunelleschi cum uno socio et cum dubus equis vadat ad castrum Castelline, Rincine et Staggie ad providendum illud quod opus est circa fortificationem castro rum dictorum loco rum expensis Opere; et ad presens camerarius Opere teneatur del pecunia Opere eidem mutuare libras decem pro expensis ytineris

1430/1 Febbraio 15 *Per andare ai castelli (castra) di Castellina, Rencine e Staggia*
Detti operai mancando i loro colleghi Alessandro di Alessandro e Bernardo di Iacopo Arrighi rispettate le opportune formalità deliberavano che Filippo di ser Brunelleschi con un compagno e due cavalli andasse al castello di Castellina, Rencine e Staggia per provvedere ciò che è necessario spendere da parte dell'Opera riguardo la fortificazione dei castelli di detti luoghi; inoltre deliberavano sul danaro affinchè l'attuale Camerario (Camerlengo) dell'Opera fosse tenuto a prestare dieci lire per le spese di viaggio

Ricerca, traduzione e impaginazione: R. Corazzi e D. Taddei

Fig. 25.

cation labor against a possible invasion of these territories (an event that actually did occur, by the troops of the captain, soldier of fortune, Niccolò Piccinino, paid by Duca di Milano Filippo Maria Visconti)[3]. From the analysis of the fortress under consideration, and of the surveys carried out by Dr. Cinzia Cosi, one can deduce that the type of construction used is the same as that of the dome of Santa Maria del Fiore. Here, too, we are in the presence of a three-leaf *('a sacco')* type of masonry.

3. See *"La Rocca di Staggia Senese"*, ed. D. Taddei, Nencini Editore, Poggibonsi 2007.

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