Reconciling Geometry and Dynamics: Models for Oval Domes

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Modern monitoring provides an exhaustive depiction of the structural health state and easing the plan of maintenance and restoring interventions on historical buildings. Dynamic investigations, in particular, contribute to the calibration of mechanical and geometrical models for seismic reliability assessment. The present paper is intended to report and discuss on a few recent experiences on reconciling geometrical survey information with the measured or calculated dynamic response of ancient heritage structures. When vibration tests are performed on the structure, acquisition records usually consist of acceleration response signals measured under ambient excitation. Reported applications will regard structures with oval shape domes, such as the Sanctuary of Vicoforte, S. Maria delle Grazie in Casale Monferrato and S. Agostino in L'Aquila.

Keywords: *geometry, dynamics, oval domes*

Introduction

The conservation, reinforcement and restoration of the architectural heritage generally require a multidisciplinary approach. The ISCARSAH guidelines (ICOMOS, 2003) define the restoration of heritage structures in a peculiar metaphor: the structural rehabilitation of heritage structures can be seen as the cure of a sick person, hence "the heritage structures require anamnesis, diagnosis, therapy and controls, corresponding respectively to the searches for significant data and information, individuation of the causes of damage and decay, choice of the remedial measures and control of the efficiency of the interventions". As a matter of facts, recent seismic standards, such as Directive (PCM 2011) in Italy, introduce and encourage the attainment of an appropriate "level of knowledge" which can be obtained solely through extensive tests and investigations. In particular, when dealing with a historic building, this must be done in several steps including: (a) Identification of the structure; (b) Geometric data gathering; (c) Historical analysis; (d) Geometrical survey of the materials and their state of preservation; (e) Mechanical characterization of the materials; (f) Soil and foundation analysis; (g) Monitoring. As for geometric survey, nowadays sophisticated and accurate techniques are available, including laser scanner, interferometry, thermography etc. Needless to say, the acquisition of knowledge meets with the difficulties normally associated with the assessment of existing structures that have to be preserved.

Dynamic investigations contribute to the calibration of mechanical and geometrical models to use at the reliability assessment stage. Moreover, dynamic testing is one of the few studies that can shed light on the global behaviour of a building. Hence, it is indispensable to the characterization of complex structures, and especially to the formulation of accurate predictions as to their dynamic and seismic behaviour. Due to the uncertainties and difficulties encountered in defining a modelling method for generalised application to masonry structures, the results of numerical analyses – performed as a rule with finite element methods (FEM) – must be reconciled with experimental evidence.

The present paper reports and comments about a few recent experiences on reconciling geometrical survey information with dynamic acquisition records, usually coming from acceleration measurements under ambient excitation. Reported applications will regard essentially ancient heritage domes with oval shape. The final product is a virtual model that is able to predict the linear dynamic response under earthquake excitation.

With its internal axes of about 37 by 25 m, the dome of the Sanctuary of Vicoforte, Italy, is the fifth biggest in the world in absolute terms, and by far the largest oval dome ever built. The project is due to Ascanio Vitozzi, and the construction began in 1596. The dome-drum system has suffered since the beginning from significant structural problems, partly due to soil settlements, and, to a considerable extent, arising from its bold structural configuration (Chiorino et al. 2008).

The church of S. Agostino in L'Aquila was built at the beginning of XVIII century, replacing an older church collapsed during the strong earthquake of 1703. The building was designed by Giovanbattista Contini, an architect of Bernini's school. Contini's plans typically take into consideration the resistance to earthquakes enough to be considered a "seismic architect". The church has a central layout and an oval dome with axes of about 21.5 by 15.5 m. During the infamous April 6th 2009 earthquake, severe cracks concerned the hall-drum-dome oval system, the apse and the bell tower suffered important damage. Moreover, the lantern collapsed and the façade was subjected to an overturning mechanism.

The elegant Baroque church of S. Maria delle Grazie in Casale Monferrato is better known by its earlier designation of S. Caterina. The church is a master-work of Giovanni Battista Scapitta, completed after his death by Giacomo Zanetti in the year 1726. The church is marked by an oval dome with axes of about 15 by 10 m, a drum 7 m tall and by a curvilinear façade. The church is also characterised by a prominent retro-choir, the dimensions of which are comparable to the church dimensions.

The three oval domes have been the object of extensive surveying operations and dynamic investigations as well, aimed at calibrating virtual models to be used in predicting the response to seismic excitation.

Monitoring systems for cultural heritage structures

Historical structures and infrastructures are inevitably subjected to ageing effects and require expensive maintenance acts and surveillance against accidental events in order to preserve them. In addition to the traditional methods, in the last three decades new experimental procedures have been developed in order to provide widespread and accurate information about the structural performance and integrity.

Farrar and Worden (2007) define structural health monitoring (SHM) as a process which involves the periodic monitoring of a structure through measurements,

Fig. 1. Sant'Agostino dome showing the effects of the 2009 earthquake.

the extraction of features symptomatic to the phenomena under investigation and their statistical analysis to determine the actual state of the system. A diagnostic monitoring system is therefore the result of the integration of several sensors, devices and auxiliary tools, like:
— a measurement system;
— an acquisition system;
— a data processing system;
— a communication/warning system;
— an identification/modelling system;
— a decision making system.

Even if it is based on innovative measuring, analysing, modelling and communication techniques, SHM shares the same goals of traditional methods. In fact, the diagnostic monitoring can be considered as an extension of the well-established investigation practices since it integrates these novel technologies in a unique smart system. SHM tries to overcome the limitations of traditional visual inspections.
The traditional survey methods are affected by a large series of technical drawbacks. Visual inspections are generally performed with a periodicity too spaced in time which risks affecting their predictive nature. Moreover, they are neither exhaustive, because they do not allow to identify hidden defects or the invisible effects of an on-going damage process, nor objective, because the estimation is related to the subjective judgement of an expert who can be fallible. More specific and accurate non-destructive testing (NDT) techniques (Shull, 2002) are carried out off-line and usually only after the damage has been located. This means that in the meanwhile an excessive level of deterioration could have been reached. Nevertheless, non-destructive estimations are performed in a local manner and so can provide useful information referred to a limited portion of the structure.
Modern diagnostic monitoring systems were born with the prerogative to overcome these limitations providing an exhaustive depiction of the structural health state and easing the plan of maintenance and restoring interventions. Recently, the vibration-based damage assessment has proved its potentialities in different applications. Modal properties have been successfully used for the damage identification in real existing structures. Many issues require further investigation and still represent challenges that have to be undertaken. A new philosophy must be pursued, which is aware of the importance of a reasoned design of the monitoring system. It must integrate a sensors network which is capable of operating a continuous surveillance and providing reliable analyses based on different information sources. The environmental and operating conditions variability must be taken into account too.

SHM Technologies

The SHM is the result of the integration of disciplines and technologies flatly different each other, but reciprocally essential. The elements which combine to bring about the conception and the execution of a reliable monitoring system can be subdivided in: experimental, analytical and information technologies (Ruocci, 2010).

Experimental technologies

Belong to this category all the techniques and the methodologies developed throughout the years to sense the structural response. Further classifications employ different discrimination criteria, for instance the monitoring goal, the static or dynamic acquisition of the measurements, the destructive or non - destructive nature of the tests, the monitoring duration, etc. Table 1 provides a brief summary of the most important experimental technologies classified according to their static, dynamic or hybrid nature.

Analytical technologies

Analytical technologies provide the tools to simulate the structural behaviour in the operational condition and in the presence of damage. The analytical investigation is essential both in the diagnostic and in the prognostic phase. In the former, the causes of damage are identified from the correlation between the observed and the analysed symptoms; in the latter their evolution is tracked. The structural modelling can be geometric or numerical, but always deterministic. Recently, a stochastic approach is spreading, in particular in the case of historical constructions, where the uncertainties referred to the mechanical properties of the materials, the boundary and connection conditions and the strong non-linear effects cannot be disregarded. However, further research

Table 1. Principal experimental technologies used in SHM.

Static test	Non-destructive static tests	Measurement of the response in a limited portion of the structure
	Destructive static tests	Laboratory or in-situ destructive tests: expensive and difficult to generalise
Dynamic test	Non-destructive dynamic tests	Vibration analysis carried out to extract modal parameters, using different excitation sources (ambient, hammer, drop of weight, electro-dynamic actuators).
	Permanent monitoring	Measuring system permanently installed on a structure, acquiring periodically different quantities.
Hybrid test	Geometric monitoring	Laser scanning, global positioning systems, photogrammetry, remote sensing technologies in order to track geometry changes.
	Non-destructive evaluation	Non-destructive technologies able to detect hidden construction details, defects or damage or to determine the physical and chemical properties of the materials.

is still required and in most of the cases a deterministic approach, whose uncertainties are handled varying the parameters within some bounds and according to the results of sensitivity analyses, is preferred.

Anyway, the adopted model must be able to capture the geometric, mechanical and boundary properties in an extensive manner and to simulate the damage scenarios of the structure in a reliable way. In this sense the experimental data represent a precious source of information to assess the model and its predictive capabilities and to calibrate it accordingly. The instruments which contribute to the analysis of the structural behaviour are:

— elaboration and representation tools (CAD, databases);
— instrumental survey technologies (photogrammetry, laser scanner ...)
— numerical modeling methods.

Computer-aided design (CAD) packages represent the first step in the analytical modelling and structural identification. A geometric conceptualisation of the construction is achieved through a three-dimensional depiction. Photographic and photogrammetric pictures can be used to define the dimensions accurately and to capture all the most significant details. The numerical modelling can be performed according to different levels of resolution and consequent computational efforts. The most commonly implemented methods are the finite element method (FEM), the boundary element method (BEM) and the finite difference method (FDM).

The FEM is generally the adopted solution because of its large spread and the availability of many commercial packages. Three-dimensional models are essential to deal with the geometric complexity of the historical constructions and to capture their peculiar loading and damage conditions. Some of the most common objectives for modelling are:

— to provide a basis for the monitoring system design;
— to provide a term of comparison for the results of the structural identification;
— to serve as a baseline for assessing any future change in the structural conditions and to define some reliable warning thresholds;
— to evaluate the vulnerability of the structure and to identify those critical elements which require a focused attention in the monitoring;

— to evaluate possible causes of observed damage, dysfunctional behaviours or performance deficiencies;
— to forecast the evolution of observed damage scenarios according to the actual structural conditions;
— to design restoration interventions on the structure.

The available methods of analysis are subdivided between linear and nonlinear. Most software packages offer a wide range of linear analyses, which include static analyses under various combinations of stationary loads, eigenvalue analysis to determine the modal properties of the structure and dynamic analyses. Non-linear analyses are characterised by the capability to take the mechanical and the geometric non-linearity effects into account. Under specific conditions, non-linear analyses allow to estimate the position of possible cracks openings and the consequences of other damage phenomena in the structure and to track their evolution in time. The drawbacks of this class of analyses are the high computational effort and calculation time, the difficulties in the results interpretation and the huge sensitivity of the accuracy of the analysis to the parameters selection. The choice of the wrong set of analysis and mechanical parameters can affects drastically the results and lead to a complete misunderstanding of the investigated phenomena.

Information technologies

The collection of the experimental data is useless if it is not efficiently supported by data recording, storage, analysis, interpretation and representation techniques. The complex process of the raw data transformation into information useful for the monitoring purposes must be accomplished in a systematic manner, adopting appropriate methodologies. Procedures for the collection, classification and storage of experimental data have to be defined in the structural monitoring design phase. The number and typology of the employed sensors, the number of the available acquisition channels and the selected sampling rate are conditioning elements for the implementation of these techniques.
Some of the most common operations, performed after data acquisition, are here summarised:
— Data formatting: signals referred to similar measurements must be formatted in the same way in order to ease the automation of the data processing.
— Data classification: the acquisition results must be organised according to a hierarchic framework and an appropriate nomenclature in order to ease the data storage and collection.
— Preliminary check: to verify the correctness of the signals acquisition and data formatting and classification.
— Data communication: from the acquisition system to a remote unit through cabled lines or wireless systems. The rapid development of more and more powerful data communication technologies will allow the spread of real-time monitoring system and will reduce their costs.
— Data storage: must follow pre-established criteria which take into account the subsequent need for signal processing, visualisation of the results and information sharing. The adoption of databases management programs and systems responsible for data storage can be helpful.
— Data pre-processing: it includes all those operations aimed at the preparation of data for the subsequent feature extraction. They are generally referred as “data cleansing” and some examples are the filtering aimed at the noise reduction, the decimation, the mean removal or the outliers removal. The data cleansing process can also involve the selection of data to pass or reject according to the knowledge gained during human inspections of the test equipment. Sometimes also the conversion of a time-history to a spectrum by means of the Fourier transformation is considered as pre-processing operation to reduce the data dimension.
— Feature extraction: consists in the extraction of useful information for the monitoring purposes from the pre-processed data. The usefulness of the extracted features is referred to the damage sensitivity. For instance, in the case of the dynamic monitoring, the acquired acceleration time-histories can be processed by means of modal identification techniques in order to extract the modal properties of the structure. These features are sensitive to the structural changes produced by damage and can be employed for its assessment.
— Information interpretation: is the formulation of the diagnosis about the structural health based on the information inferred from the experimental data. Several statistical tools can be used to accomplish this task. The more common are multivariate correlation analyses, bayesian methods, pattern recognition methods, neural networks and genetic algorithms. The preliminary com-

parison with the results obtained from the analytical simulation is worthy too.

— Results presentation: is the synthesis of the information obtained from the monitoring and it depends on the nature of the results and the pursued goals. The results are generally portrayed by means of plots which show the evolution of the extracted features or through indices assumed to be sensitive to the structural health. The simplicity and the immediacy of the representation are essential requirements.

— Decision making: is the conversion of the results into a decision whether any action need to be taken and, eventually, what kind of action should be.

Description of Three Oval Domes

Dome of the Sanctuary of Vicoforte

The Sanctuary of Vicoforte, a bold, highly prestigious structure, with its centuries-old history and the damages suffered in the past, is a classic example of a cultural asset exposed to earthquake hazard, being located in the proximity of a seismic area. Since the earliest stages of construction, the building was adversely affected by important differential settlements of the foundations, due to an unfortunate selection of the site. The dome-drum system has suffered over the years from significant structural problems, partly due to further settlements of the building induced progressively by newly built masses, and, to a large extent, arising from the bold structural configuration of the dome-drum system itself.

In 1983, concerns over the severe settlement and cracking phenomena affecting the structure prompted the decision to undertake inspection, monitoring, and strengthening interventions. After a survey and investigation campaign a strengthening system was put in place (1985–1987). It consisted of 56 active tie-bars placed within holes drilled in the masonry at the top of the drum along 14 tangents around the perimeter, slightly tensioned by jacks. A monitoring system was set up to measure strains and stresses in the structure and crack propagation, as well as stresses in the reinforcing tie-bars (re-tensioned in 1997).

Research and monitoring activities are still in progress on this building (Chiorino et al., 2008), including the realization of a non-linear model for the whole structure-foundation-soil system. Because of the technical complexity of the masonry structure of the building and the peculiarities of the soil on which it rises (presence of deep layers of soft soil, which caused appreciable settlements over the years and in the event of an earthquake would probably have significant effects on seismic input), the acquisition of knowledge and the modelling process pose difficulties to be addressed through an all-encompassing approach (Calderini et al. 2006; Chiorino et al. 2008). Several studies concerned the geometry of the complex structure (Novello and Piumatti, 2012; Novello and Piumatti, 2012). Dynamic studies have been also conducted on the Sanctuary, designed first of all to assess seismic risk, while they also provide an opportunity to test out the application of the new regulations (Chiorino et al., 2011). In fact, the Sanctuary of Vicoforte has been recently chosen as a case study for the evaluation and application of the Directive PCM 2008, in the frame of an agreement protocol with the Italian Ministry for Cultural Heritage. Although Vicoforte is characterized by low seismicity, the seismic vulnerability of the cathedral deserves to be investigated owing to its historical, architectural and structural significance.

Earlier studies conducted on the Sanctuary of Vicoforte included investigations aimed to characterise the masonry and the foundation soil, and to determine the geometric data and crack patterns (for further information on the construction of the Sanctuary and the investigations conducted previously, see (Chiorino et al., 2008). The next step along these lines was the execution of non-destructive dynamic tests, designed to characterise the dynamic behaviour of the structure and the mechanical properties of the materials, and to identify the overall and local response of the structure.

Dome of S. Agostino church in L'Aquila

The eighteenth-century church of S. Agostino, was rebuilt after the 1703 earthquake destroyed the pre-existent medieval church. The church is located in the city centre of L'Aquila, contiguous to the more known Palazzo del Governo, almost completely destroyed by the recent earthquake of April 2009. A series of tests conducted on the foundations showed that the structure is completely independent from pre-existent buildings. The most recent intervention on the church was the re-

Fig. 2. S. Agostino church: (a) before earthquake, (b) after earthquake.

pairing of the lantern roof: the old cover was substituted with a new metallic one, linked to the original stone columns using a concrete ring.
The church is composed by three parts: the atrium, the body and the presbytery, closed by a circular apse. The body is surmounted by an impressive oval dome. It is worth notice that this dome is built directly on the piers, without a drum. Nevertheless the oval dome presents an octagonal dome cladding (*tiburio*) which has not structural role. In correspondence of the vertex of the octagonal cladding, there are eight buttresses, linked directly to the dome through masonry walls in correspondence of the internal pilaster. The dome is therefore subdivided by these walls in eight groins and it has a wooden roof supported by the buttress walls themselves.
The earthquake of April 2009 severely damaged the church. The dome-*tiburio* system was heavily damaged and showed in plane shear failure (diagonal-cracking) in each groin of the dome and on the walls of the octagonal *tiburio*. The lantern was completely destroyed by the earthquake, whilst the bell-tower was heavily damaged. The first structural survey (Calderini and Lagomarsino, 2010) detected also a tilting mechanism of the façade and cracks in the apse, the presbytery and in the lateral chapels. The annexed bodies were damaged also, such in the case of the sacristy vault.

Dome of S. Caterina in Casale Monferrato
The origin of the church is a donation of the marchioness Anna d'Alençon to the Dominican Sisters of S. Caterina da Siena, occurred in 1528. The religious congregation relocated in the new palace and built an oratory (the current choir). In eighteenth century the Sisters wanted to renew the church, so they commissioned Giovanni Battista Scapitta to design the extension. The new church was placed side by side to the old oratory and was built in 8 years. Works finished in the 1726.
The body of the church is mainly characterised by an oval dome built on a 7 meters tall drum. The dimensions of the dome are 15 m by 10 m, with a height of 4.5 m. The building includes also a small atrium and an apse (communicating with the oratory). The dome is formed by 8 masonry groins and as many buttresses. The interior of the dome is richly frescoed, though deterioration is in progress.
The dome presents a slender lantern 5 m tall with 8 pilasters, one for each buttress, recalling the main structure. The roof of the lantern is again a little masonry dome. The richly decorated Baroque façade has a jut which exceeds the height of the church body of 6 m.
The old choir has dimensions in plant of 10 m by 22 m and it is covered by a barrel vault, which is supported by regularly distanced round arches in correspondence of the pilasters and is reinforced with metal ties.

Fig. 3. S. Caterina church in Casale Monferrato: (a) Dome and the façade, (b) front view of the façade and the pinnacles, (c) interior view of the dome.

Virtual Models

Geometric models

Geometric models obtained from survey activities for mechanical representation of a structure usually do not need to be too detailed. In fact, mechanical models are computational intensive; therefore the geometric model should not contain data that are useless to the solution of the mechanical problem. Moreover, when geometric information is assimilated by finite element (FE) mechanical models, the required homogenisation of material properties nullifies any possible improvement. Conversely, other information from survey may become extremely important, such as type of contact/interaction between different blocks/bodies, presence of cavities or dislocations, remote parts and components etc.

The geometric data related to the Sanctuary of Vicoforte were obtained from an examination of the building performed with a laser scanner by the Nagoya City University research team coordinated by Aoki (2004), and from surveying measurements. It is worth noticing that in the case of the Sanctuary of Vicoforte the accuracy of the geometric data is important due to the settlement history that occurred to the building. Therefore, the laser scanner technique is probably the most straightforward technique to build a 3D geometric model of the church which can be reliable for structural modelling.

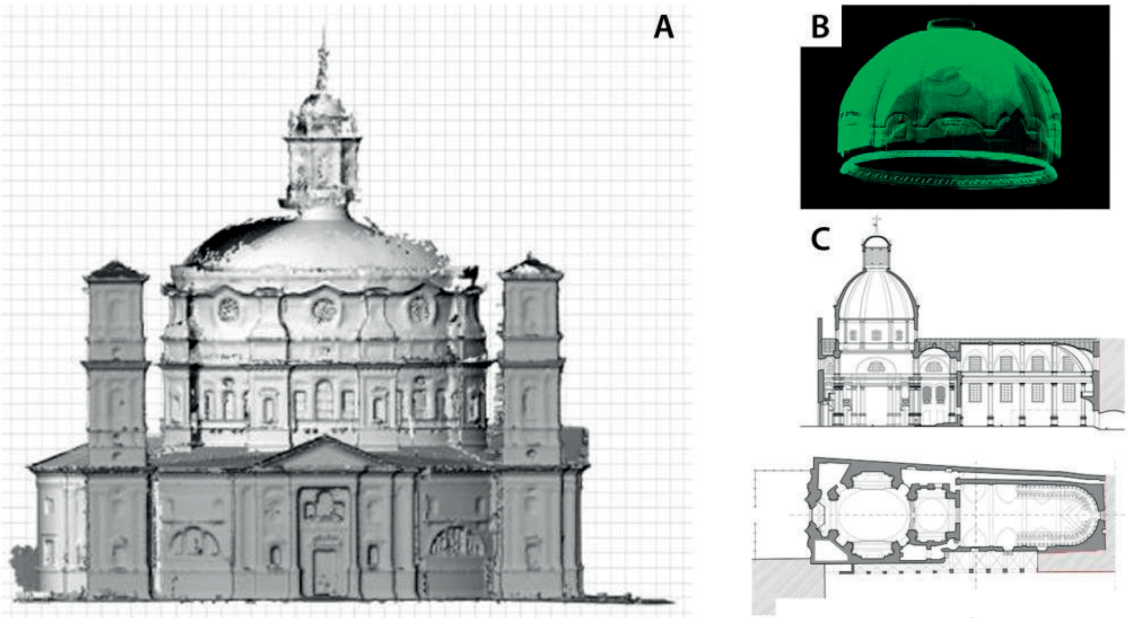

Fig. 4. Three different types of representation for the three domes: (a) Lateral view of the dome of the Santuario of Vicoforte, compared with the rest of the structure, from a laser scanner survey (b) Cloud of points of the dome of Sant'Agostino extracted from its context, from a laser scanner survey, (c) Celerimetric mapping of the S. Caterina church with its choir.

For what concerns the S. Agostino church, laser scanner data of the dome were collected by the Padova University research team, coordinated by Achilli (Calderini and Lagomarsino 2010). In order to reconstruct the complete geometry of the church, in this case, the laser scanner data were not exhaustive. Inspections conducted by the fire brigade (VVFF) after the 2009 earthquake allowed finding unknown structural elements, such as the eight buttresses linking the dome to the *tiburio*, and filling on the top of the dome, whilst other elements were modelled using old geometric surveys. These data were cross-checked with the laser scanner data showing a discrepancy of almost 40 cm between dome surface between the new and the old survey.

On the other side, the geometric model representing the S. Caterina church in Casale Monferrato was built mainly using pre-existing geometric surveys. A few cross-checking were performed using laser measurements of the most relevant dimensions, such as the dome height. Also in this case important discrepancies between old surveys and new measurements were found. The errors are seen especially in the building height measurements, this meaning that in the past vertical measurements were harder to assess.

Mechanical models

The aforementioned geometric surveys were firstly used to build solid geometric models. The geometric virtual models were used as first input in the realisation of finite element models of the structures. In all three cases the finite element package ANSYS was used to build a linear model of the churches.

The cracks formed in the structures were not taken into account in constructing the FE models, based on the assumption that their effects on the order of presentation of the natural modes of the structure are virtually negligible.

The Sanctuary of Vicoforte and S. Caterina church FE models have been updated using vibration measurements, which were available from specifically designed dynamic tests. The model updating procedure (Friswell and Mottershead 1995) consists in performing an optimisation procedure of a cost function which compares analytical frequencies and/or modal shapes with experimental frequencies and/or modal shapes. This allows updating the mechanical properties of the structure materials, such as Young's modulus or Poisson's ratio.

The FE model realisation of the Sanctuary of Vicoforte and the model updating procedure are summarised

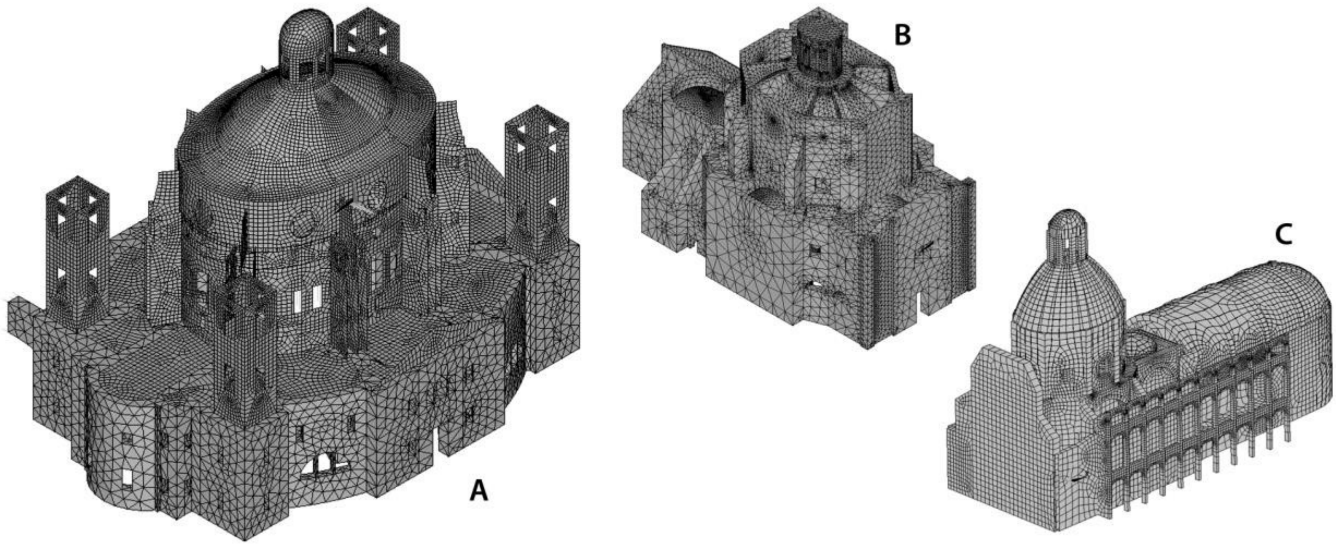

Fig. 5. Finite Element Models of the three churches (figure out of scale): (a) Sanctuary of Vicoforte, (b) Sant'Agostino, (c) S.Caterina. The figure shows the three FE models realised. As it is possible to notice, only the most significative structural elements were represented in the geometric models. For instance, the wooden roof of Sant'Agostino and S. Caterina has been discarded because it is irrelevant from a structural point of view. Clearly enough, the mass of this element has been conveniently distributed on contiguous elements.
One of the main tasks in the realisation of a FE model is the meshing. Masonry structures are usually meshed using three-dimensional elements due to lack of a prevalent dimension. In all three reported cases a solid element has been chosen to mesh the model, using tetrahedral elements in the first two cases, and brick elements in the latter.

in a paper by Chiorino et al. (2011). The same article describes also the dynamic testing campaign, which interested only the dome, as well as the various stages followed for the modal characterization of this complex structure.
The model updating procedure, performed essentially on the natural frequencies, allowed updating the masonry mechanical characteristics as reported in Table 2.
For what concerns S. Agostino, as stated before, the building is part of a complex; therefore the boundary conditions on two sides of the building had to be carefully modelled. The main defence against earthquakes are the masonry walls of the adjacent *Palazzo del Governo*: these elements, orthogonal respect to the church, possess great inertia; therefore they have been modelled as constraints on the eastern side of the church.
Finally, the FE model of the S. Caterina church represents the church body, the façade and the dome. The adjacent choir and buildings have been firstly modelled in two ways: assuming the contiguous bodies as rigid (5000 MPa) or as soft (1000 MPa), but the two assumptions showed similar results. As in the case of the Sanctuary of Vicoforte, the model has been updated using the vibration measurements collected during an experimental campaign (23rd-29th September 2010). The model updating procedure was realised using a MatLab®

Table 2. Sanctuary of Vicoforte: updated parameters of the FE model and comparison with sclerometer experimental data reported by Aoki et al. (2004).

Property:	First attempt value:	Updated value:	Test value (2004):
Elastic modulus, E:	1635 MPa	2330 MPa	1300 - 4800 MPa
Poisson's ratio, ν:	0.4	0.38	0.39

toolbox implemented at the Dep. of Structural Engineering of Politecnico di Torino, SDIT3, which allows the interoperability between the FE model and the experimental analysis.
In the case of S. Caterina, visual inspections and historical analysis allowed distinguishing four different masonry macro zones: one for the church body, one for the dome-drum system, one for the lantern and one for the façade depending on age of construction and deterioration. The macro zones have been individuated as regions with homogeneous mechanical characteristics, also based on historical data. Table 3 shows the result obtained through the modal updating. As it is possible to notice, the material of the dome-drum system and of the church body seems to be of better quality if compared to the masonry of the façade and of the lantern.

Prediction ability of reconciled models
The model updating procedure illustrated in the previous section is an important result per se; in fact it allows estimating the mechanical properties of materials. Nonetheless, the final aim of the dynamical studies of historic structures is the realization of a predictive analytical model for seismic protection.
These predictive models, even if linear (as in the three reported cases), can be helpful in the design of structural monitoring systems allowing an improved sensor positioning on the monitored structure. For instance, in the case of a dynamical structural health monitoring, the sensors must be collocated using sensitivity analysis respect to modal shapes. In other words, one has to find the optimal setup of sensors that allows detecting the maximum number of modes using the minimum number of sensors.

Table 3. S. Caterina church: updated mechanical parameters.

Element	Elastic Modulus [MPa]	Density [kg/m^3]	Poisson ratio
Church body	3625	1733	0.40
Dome-drum	4500	1733	0.40
Lantern	1025	1733	0.30
Façade	1250	1733	0.40

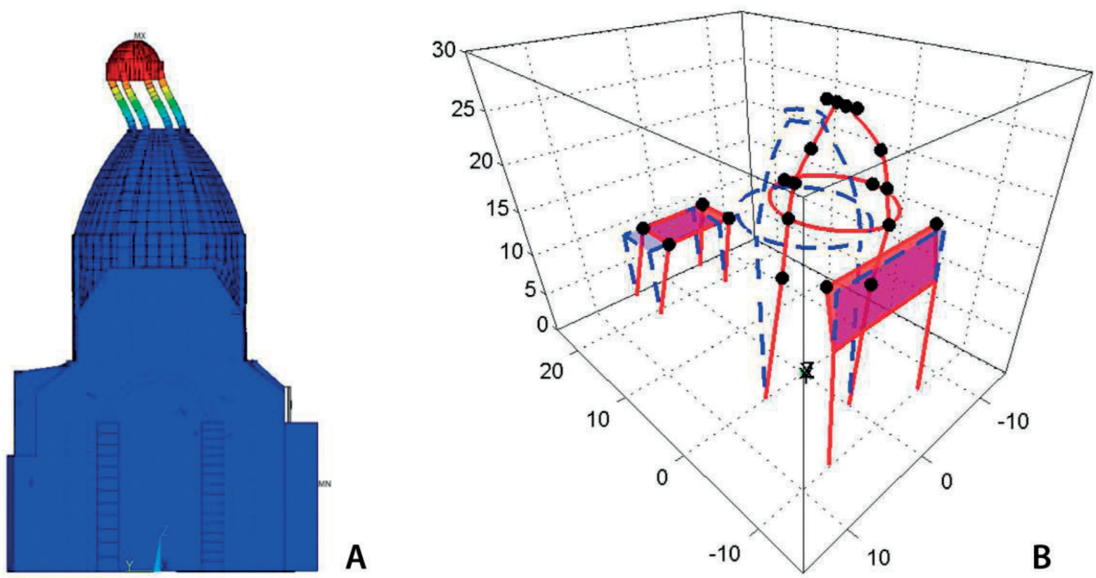

Fig. 6. S. Caterina church: FE modal shape (left side) as compared to the experimental one (right side).

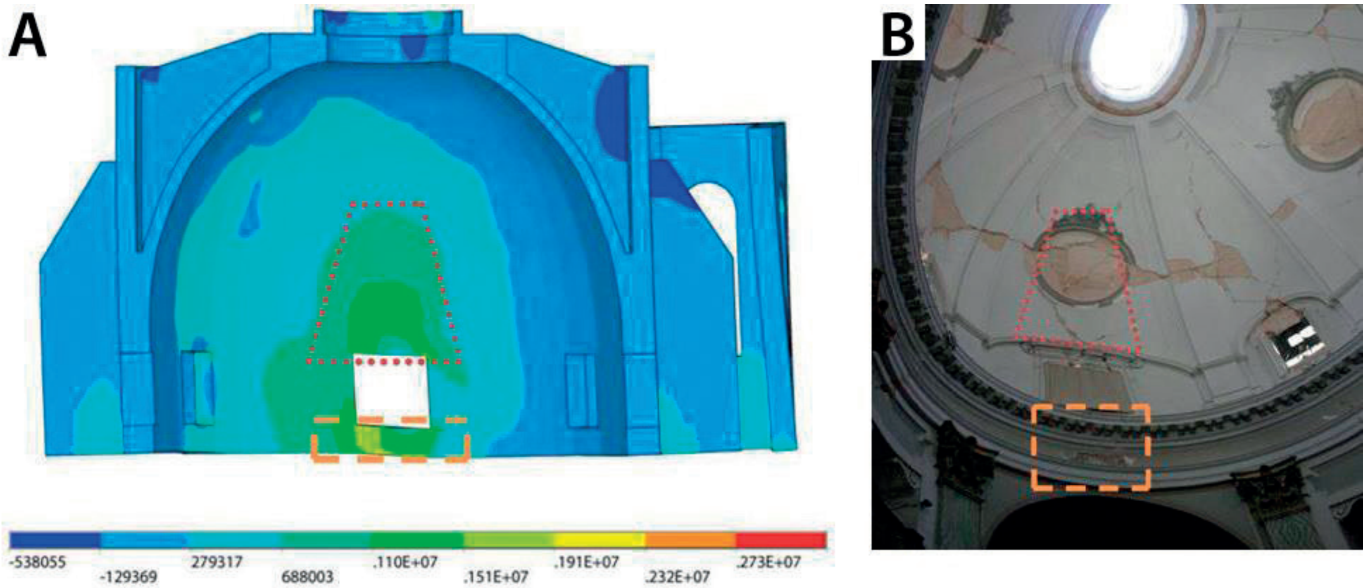

Fig. 7. S. Agostino church: comparison between stress concentration in the FE model (a) and real damage in the dome groins (b).

In some circumstances, even linear FE models can predict the possible damage produced by intense seismic motions. Indeed, in the case of S. Agostino church in l'Aquila, the crack pattern caused by the ground motion is easily found using the facilities of time history FE analysis. Fig. 7 shows FE stress concentrations associated to seismic measurements recorded during the 6.3 Magnitude earthquake that hit l'Aquila, on April 6, 2009, 3:32 AM, local time, causing major damage and hundreds of deaths. Accelerograms used in these simulations refer to the basement of Forte Spagnolo. The linear FE analysis in this case appears to retains a predictive capability with respect to the damage actually suffered by the monument (comparison in Figure 7), which apparently has been induced by the characteristic dome stiffeners designed by Contini.

Conclusions

This paper has reported an overview of the problems concerning geometric survey of cultural heritages for structural modeling. The deep connections between geometric survey and structural health monitoring are highlighted in all his forms through three notable examples of oval domes. In both the cases of the Sanctuary of Vicoforte and of S. Caterina in Casale Monferrato dynamic measurements were used to perform a model updating procedure. The calibrated models have been used for the evaluation of the seismic response of these monuments, also in view of future interventions on the structures. On the other side, the S. Agostino model allowed to investigate the damage mechanisms activated by the 2009 earthquake in L'Aquila, showing the great importance of stiffeners in the seismic response of the domes. The demonstrated capability of these defense lines in activating a more ductile, as well as non-thrusting, structural scheme seems to confirm Contini's reputation.

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