3D digital models of Brunelleschi's Dome for geometrical analysis

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Brunelleschi's Dome in Florence is one of the world famous Italian masterpieces, a prime example of Renaissance vaulted constructions of remarkable dimensions, built according to tradition without centring thanks to unusual building techniques. The subject of several research and survey studies over the centuries, the structure has been analysed especially from the geometrical point of view, in order to study in depth the proportional system and the genesis of the pointed arches of its distinguishing outline. Nowadays, thanks to the outstanding developments of technology and to the three-dimensional laser scanner devices for a detailed data capture of reality, there are new possibilities for updating the knowledge of important monumental buildings. This research presents the results of an ongoing study where reverse modelling applications play a crucial role even if not developed for Cultural Heritage, but appropriate if working with both numerical and mathematical models. In this way, an exhaustive study and check of the geometrical hypothesis on the Dome's design can be carried out in a broader framework than single sections.

Keywords: *Brunelleschi, Dome, Geometrical Analysis, Survey, 3D Digital Model.*

Introduction

The dome covering the church of Santa Maria del Fiore in Florence is one of the world famous masterpieces of the Italian Renaissance, famous both for its outstanding dimensions and for the peculiar building technique adopted to construct the huge structure without any temporary wooden supporting falsework.

Built during a quite short period – less than 16 years – the whole vaulted complex was finished in 1436, with the only exception of the lantern.

It is a well-known fact that only Filippo Brunelleschi, excellent goldsmith and clockmaker, was able to provide a solution for the greatest project of his time, thanks to his additional competence also in the field of engineering and architecture.

The main problem he had to solve was related to the octagonal shape of the drum, obviously in a formal relationship with the nearby Baptistery of San Giovanni, but considerably irregular due to several changes to the project before completion and to construction itself. In fact, between the longer and shorter interior side of the octagon there was a difference of approximately 62 cm (Corazzi-Conti 2011:96), a circumstance that entailed a greater complexity to follow the proportional structure conceived for the dome by the city jury.

With regard to the silhouette of the dome, the predecessors had chosen a pointed arch profile, following for the intrados the rule of the pointed fifth (*sesto di quinto acuto*), that is to say that the centre of the circumference of the corner arch had to be located at a fifth of the distance between two opposite vertices of the interior octagonal base. On the contrary the external shell, protecting the dome from atmospheric agents, had to adopt the rule of the pointed fourth arch (*sesto di quarto acuto*), moving the centre of the circumference at a quarter of the distance. Doing so, the final aesthetic outcome of the dome would have been more magnificent and puffed up in the Florentine skyline (fig. 1).

As further proof of the fact that for the whole city this matter turned out to be very important, above all because the respect of the conditions required for the design was

Fig. 1. View of Brunelleschi's Dome from the bell tower of the church. Photo by the author.

essential when dealing with a long-lasting building that could likely keep busy several generations of workers, in the renowned parchment by Giovanni da Gherardo da Prato (1425-26), Brunelleschi was criticized harshly for the profile still under construction and accused of not building the dome correctly.

As a matter of fact, at the time the definition of the curvature occurred by means of elementary arithmetical and geometrical reasoning, besides grid-based composition schemes and using scale models.

Antonio Manetti, Brunelleschi's biographer, tells that he built many models but in secret, so that he could keep the lid on his brilliant ideas, not to give away information to his professional enemies.

Another piece of news given by Giovan Battista Gelli reported that the architect had drawn the sections of the dome on the banks of the Arno River, probably full-scale and in a flat area levelled for this occasion, in order to better verify the curvature of the arches (Fanelli-Fanelli 2004: 29).

Therefore when studying Brunelleschi's Dome, scholars often dwell upon the geometrical genesis of the structure, analysing the composition especially from this point of view (Sanpaolesi 1977; Rossi 1992; Gurrieri-Acidini Luchinat 1994; Di Pasquale 2002; Corazzi-Conti-Marini 2005; Ippolito-Peroni 2007; etc).

If it's a given that this huge structure cannot be regular anyway, thanks to modern high detail 3D laser scanner surveys and recent developments of software solutions in the field of reverse modelling, it is nowadays possible to analyse in depth and compare reality with a conceptual geometrical model created on the base of the original design, estimating the value of this deviation not only for single vertical sections but also for whole surfaces, a fact that was impossible even in the recent past.

This paper intends to prove that a critical use of these up-to-date technologies in the field of Cultural Heritage is presently essential to achieve appropriate results, above all considering the enriched interactive toolkit available for researchers, together with latest procedures of digital survey by means of range-based devices. Hence, some possible applications on the dome's intrados are shown as case study.

Survey by laser scanner devices

One can say that survey is a tool of primary importance for documentation and knowledge of the architectural heritage, a graphic support used to represent the morphometric aspect and the state of preservation of a monument in a particular moment, before any project of conservation, reuse and management of the building.

In the course of time, several surveys were carried out on the Dome, but only with the latest active sensor devices it is possible to achieve a metric survey to the purpose of precisely showing the exact spatial shape of the vaulted structure.

The latest digital devices allow carrying out metric surveys with increased reliability, especially thanks to developments of laser scanner technologies and their applications in the architectural and archaeological field.

It is thus possible to update the outdated surveys of the dome, to have at a researcher's disposal more reliable measurements than in the past and to achieve accurate polygonal mesh models of existing architecture as close as possible to the real building.

In fact, this kind of measurement, among the many advantages, allows a better accuracy of captured data; the following step consists in converting the point cloud into a detailed mesh representing the irregularity of the real architectural element.

In this sense, this information can be useful from the point of view of static surveys for an overall systematic description of the cracks wich have occurred in the dome and for the possibility of up-to-date analysis regarding the surfaces of the vaulted sections (for example a monitoring on seasonal deformations).

Regarding the Dome of Florence, the most recent surveying activities carried out date back to 2004 (A.B.C. General Engineering in collaboration with Codevintec) and 2006 (Geoarte S.T.A.), under the scientific guidance of the University of Florence, with two laser scanner campaigns by means of time-of-flight devices to gather the entire intrados and the north-eastern area of the extrados of the double shell (fig. 2).

Thanks to this point cloud of the interior soffit, it has been possible to create a reliable polygonal mesh model aimed at studying the proportional pattern and achieving new conclusions on its geometrical genesis.

3D digital models and their potentiality

Nowadays through innovative computerised modelling tools, we can create three-dimensional digital models of different kinds, with a variable level of detail for a multiplicity of purposes.

Generally speaking, unlike physical scale models, digital ones are really more flexible, concerning the possibility to undergo modifications, to be more easily portable and disseminated, in addition to the advantage of real-time visualization that makes the interaction with the virtual space easier also for non-experts.

However, regardless of the software used, the construction of 3D digital models is not automatic but the result of the user's knowledge and skills and the critical use of digital technologies.

The creation of digital models can be based on different data sources, gathered from reality or idealized, that is to say based on 3D point cloud and mesh model or on conceptual models based on the ideal geometric genesis.

In addition, there are more and more tools for the data management and handling. For instance, the reverse modelling tools are specifically designed for the conversion of 3D meshes from active and passive sensors into mathe-

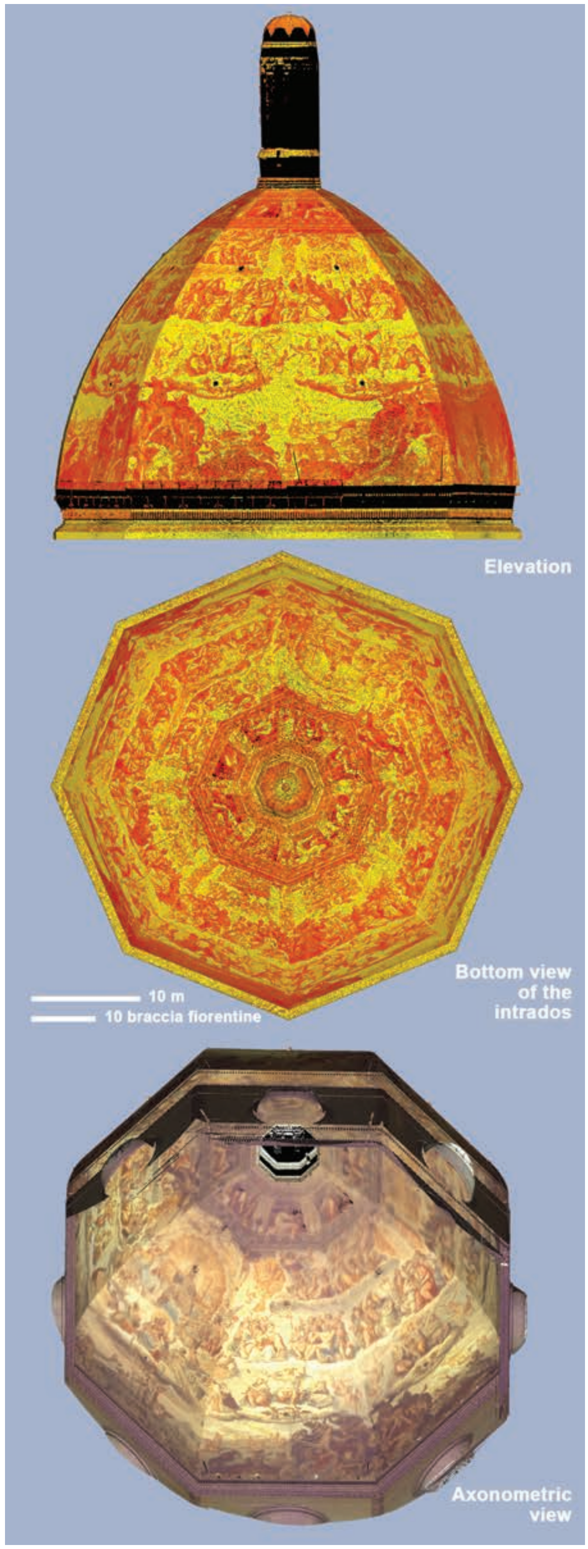


Fig. 2. Point cloud from the 3D laser scanner survey campaign of the intrados, carried out in 2006 by the company Geoarte S.T.A.

matical models (NURBS or solids). Their use is widely known and developed in the field of design, mechanical and civil engineering since these application areas need a strict control of the 3D model accuracy.

In the field of Cultural Heritage (archaeological remains, historical buildings), the three-dimensional mesh is preferable to mathematical models for dissemination practices, since it provides efficaciously an accurate texturing, optimization and light simulation rather than achieving conceptual shapes through parametric patches.

In other words, reverse modelling applications are mainly employed for point cloud triangulation, mesh healing, big files decimation, segmentation of complex models into smaller elements for an easier use and so on, but their more performing interactive tools for slicing the model (sections or feature curves and surfaces) are often underused, although very suitable when concerning shape analysis (Fantini 2013).

As a matter of fact, highly detailed meshes are more effective when an accurate representation of the system is required, even though irregular, e.g. for structural deformations in order to quantify the cyclical movements of the cracks and fractures, detachment and loss of material from the surface, thermal fluctuations due to weather conditions.

On the contrary NURBS mathematical models' strong point is the possibility to represent continuous smooth surfaces, with nice curvature flow.

In short, on the one hand there are mesh models, easy to manage in a multiplicity of applications, ranging from entertainment software to reverse modelling applications for data extraction.

On the other hand reverse modelling applications are considered more fitting with the redesign process, in which, the main outcome are NURBS or solid representations of scanned objects belonging to the field of design.

Frequently the majority of operators simply extract poly-lines and spline curves from point clouds or highly detailed meshes and then use them in computer-aided design applications, generating 3D digital models by loft, sweep, extrusions, and curve networks tools.

Reverse modelling applications are indeed the better performing and all-embracing solution to explore and obtain curves and nurbs patches from high-poly models, since they provide several aspects for a comprehensive and reliable exploration of a shape: tools for the creation of reference planes, vectors and points; tools for extraction of poly-lines and splines using reference geometries or their instances in an interactive way; tools for extracting 2D and 3D primitives (circles, ellipses, lines, spheres, planes, etc.) using high-poly meshes as reference templates; tools for the control and measuring of section deviation between point-clouds, high-poly meshes, NURBS surfaces. For this reason it is important to point out that software developers in the framework of reverse modelling applications implemented both documentation and re-design tools.

Starting from this data by means of solid modelling it is thus possible to create conceptual models – necessarily simplified for easy management – approximating the existing surfaces of the real Dome's intrados.

Referring to the Florentine Dome, the numerical model was created using INUS Technology Rapidform XOR3 (fig. 3).

The model can be interactively examined and any cutting plane, exactly located using mesh features as reference, can generate single or multiple sections (fig. 4), with the possibility to compare the regular idealized shape with the real one.

But first of all, it is fundamental to treat defects of massive point cloud conversion into mesh representation by correcting topological errors through triangle edge resampling, in order to have an isotropic mesh within the tolerance limits of the scanner accuracy.

Reference planes and corresponding sections can be divided mainly into two different categories, depending on the future use.

The first sections – polylines or curves – are used for documentation scope, since they represent a real object in a reliable way (they are composed of a large number of vertices), but they are not suitable for modelling an ideal version of an architecture. The second kind of section is useful to re-draw directly on the mesh using the latter as reference.

In reverse modelling software CAD style commands (lines, circles, ellipses and other primitive elements) are available: all these geometrical shapes are drawn using template reference sections from mesh models.

Even if reverse modelling commands are addressed to mechanical engineering and product design, they can also be suitable for Cultural Heritage representation, first and foremost because the research of shapes anchored to the mesh is simplified.

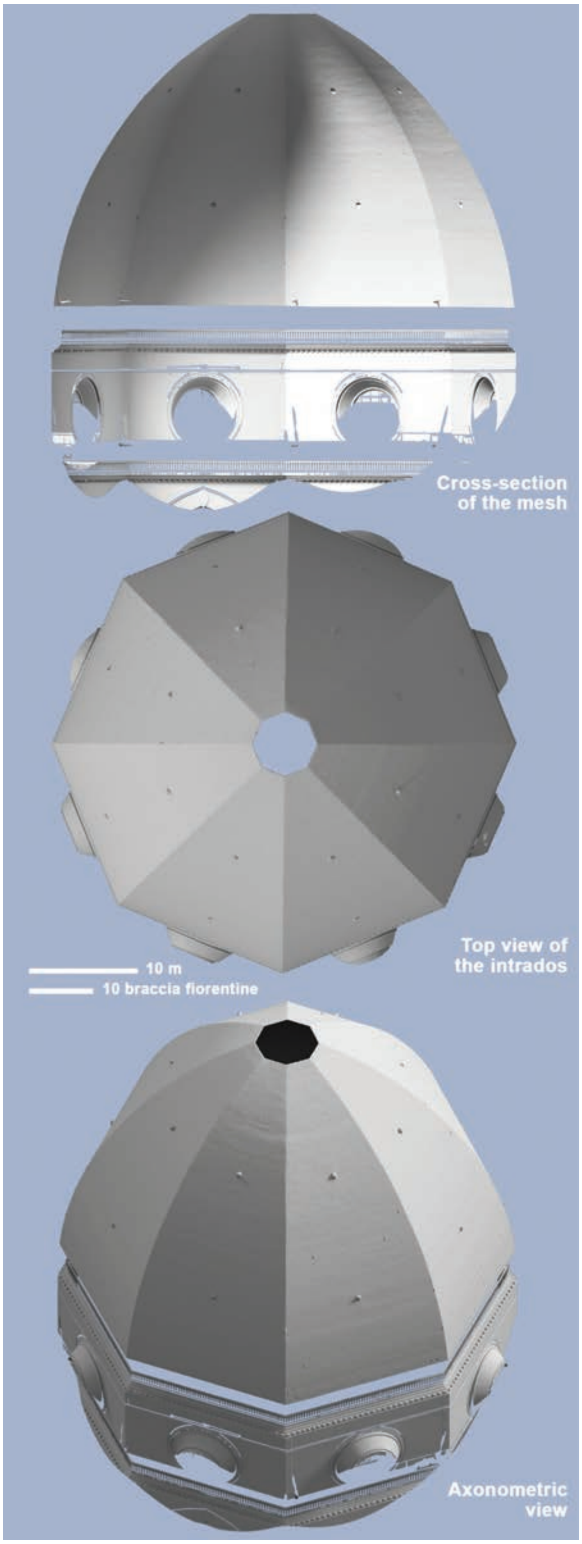


Fig. 3. Numerical model of the dome's intrados generated from the point cloud of the 3D laser scanner survey. It is worth noting the high detail of the mesh.

In fact, the automatic interpolation makes easier and faster the research of best fitting circles on the interior corners of the dome and all the sections are already positioned in a 3D environment (fig. 5).

The interesting results achieved using proprietary software during former research, were mainly focused on the analysis of specific sections obtained by laser scanner survey (Corazzi 2013).

The limit of that approach can now be overstepped by extending the analysis to the whole Dome's surface and not just considering corner sections or intermediate profiles of octagonal sectors (fig. 6-7).

On the contrary, thanks to these tools for 3D drawing it is even more convenient to consider the geometry in its entirety, because of the accurate control of a multitude of spatial information. As an instance, by means of the interactive selection it is possible to do an estimate of the deformation events of such a big dome.

Furthermore, the primitive curves obtained (best fitting circles) can be directly used as a template for modelling (especially in NURBS and solid modelling applications).

Contour lines can be used to determine irregularities on the wide surfaces of the vaulted sectors, certainly not appreciable considering only 2 or 3 reference sections (fig. 8).

Among the numerous hypotheses made of the theoretical proportion that rules the Dome's geometry, the most recent studies on the geometrical genesis are based on sections gathered from the 3D point cloud from the digital surveying campaigns (Corazzi-Conti 2011). According to this study it seems that actually the dome geometrical system is based on circumferences and ellipses.

In particular, the genesis of the eight sections starts from the drawing of an arc of circumference on the diagonal side of the base octagon, following specific rules. In this way the section on the corner is represented by a portion of circumference.

On the contrary, when sectioning in the middle of the vela a part of an elliptical arc is obtained.

Due to this assumption, the surfaces of the eight sides are determined by horizontal lines (generators of the cylinder) that move along an elliptical path, visible in the mid-section of the side, and in theory the vela should overlap on an elliptical cylinder (Bertacchi 2010).

But when using deviation tools of reverse modelling software, it is evident that the overlapping of the concep-

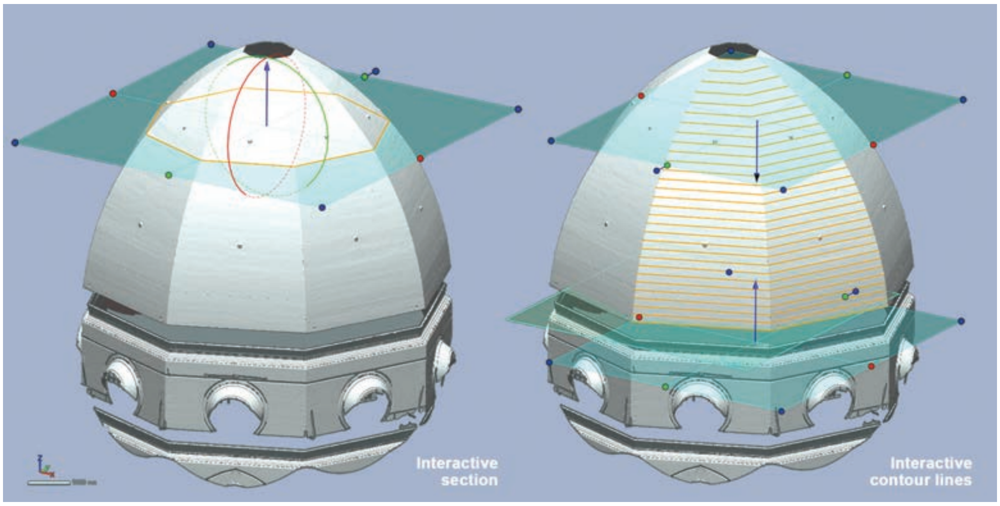


Fig. 4. Sections and contour lines determined by means of interactive tools of reverse modelling software.

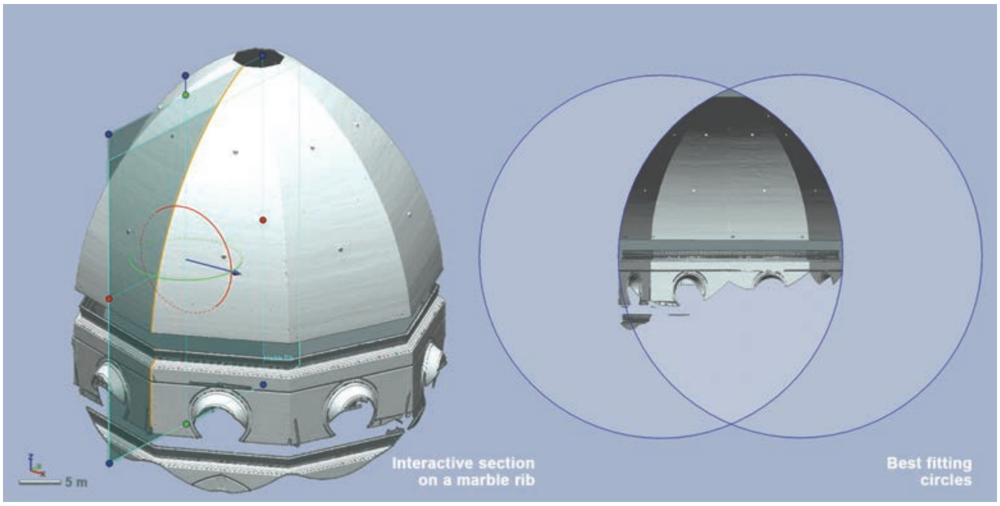
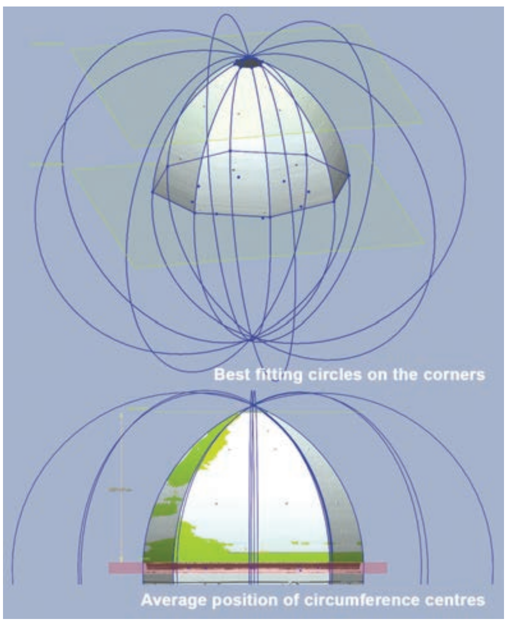


Fig. 5. Interactive section on a corner where two vele intersect and best fitting circles from automatic interpolation.



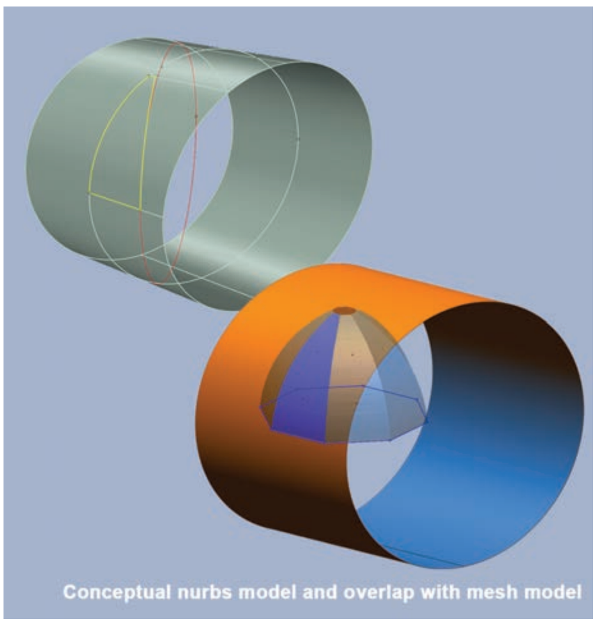


Fig. 6. Best fitting circles on the intrados corners where two vele intersect and average position of the eight centres of circumference (in red).

Fig. 7 . Nurbs model generated with McNeel Rhinoceros 5.0 and overlap with the polygonal mesh by the survey (violet).

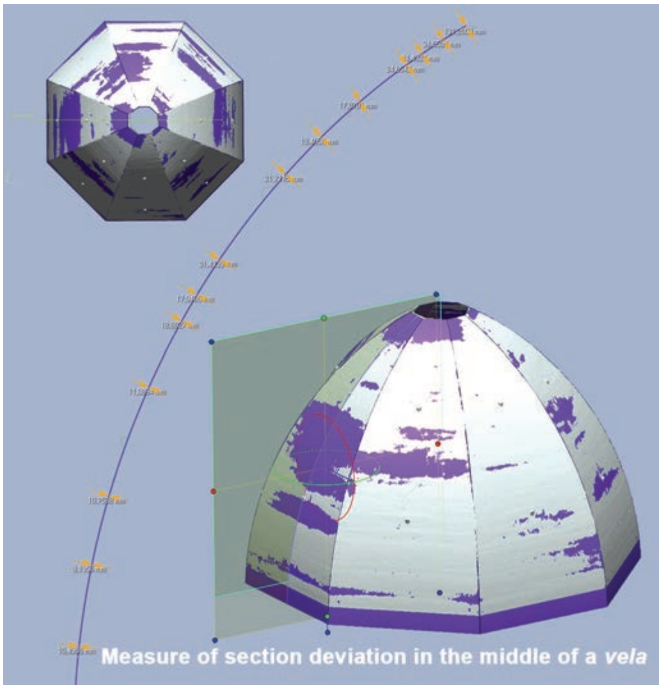


Fig. 8. Measure of section deviation in the middle of a vela and pertaining values in millimetres.

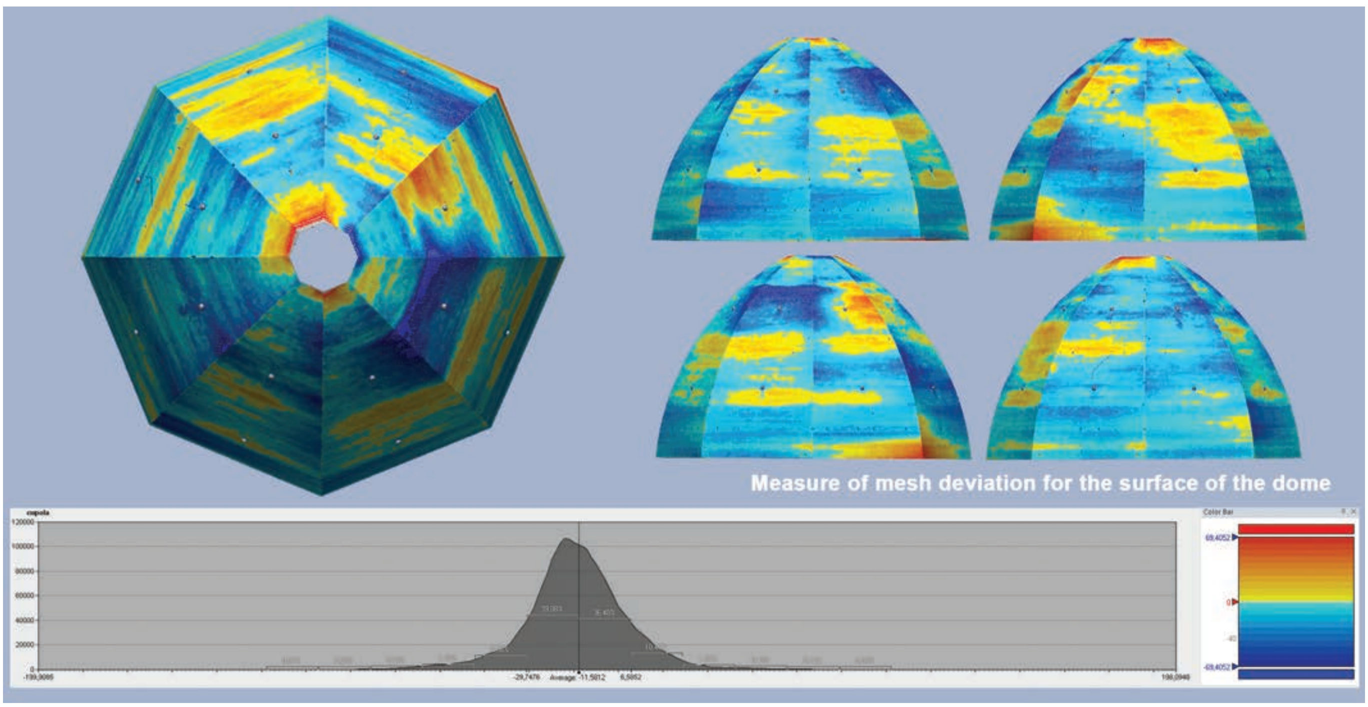
Fig. 9. Measure of deviation between the numerical model of the dome based on the 3D survey and the mathematical model (conceptual nurbs model) generated from the best fitting circles of the corner sections.

tual model on the point cloud cannot be perfect, as shows the colour range and schematic table (fig. 9). In the image there is a summary of the procedure to calculate the deviation measure between the highly detailed mesh resulting from the point cloud and the idealized model.

Best fitting circles are used for the generation of eight NURBS patches, obtained using simple loft operations, then compared to the original mesh from the scanner device.

The radii of best fitting circles, automatically obtained using a reverse modelling application, are listed in the following table.

Circumference	Measure (mm)
1	36595
2	35932
3	36000
4	36584
5	36702
6	36634
7	36123
8	36028
Average	**36324**



Surface-to-surface non-linear displacement values are mapped with a colour range, in blue for the negative deviation and red for the positive one. The colour key, for the extension on the whole Dome, shows very clearly the lack of uniformity of the deviation. This kind of visualisation allows a better understanding of the 3D models of such a huge object.

In the case of a conceptual model it is important to establish a bias, a threshold between a simplified or idealized model of a real architecture and its fully detailed representation. In this case the average deviation is equal to -11,5812 mm, while the 75,56% of the samples presents errors included between a value of -29,7476 and +6,5852 mm.

The construction of the dome's model is then a complex matter, due to the different nature of the input data, to its morphological detail and, last but not least, to the planning and management of the modelling process itself: complex shapes, including the exterior and the interior ones, as well as the structures of the dome needed to undergo a difficult segmentation phase aimed at organising in a hierarchy all the parts forming the model.

Conclusions

The kind of visualization tools used in this paper for inquiring into the deviation between a mesh model from a scanner and its idealized version are widely used in reverse modelling applications. In the design and production fields are they used for checking the existing deviation between solid CAD models and real objects produced by means of CNC machines: the accuracy of construction is a fundamental issue for industrial standards, but also for this kind of research in Cultural Heritage.

Novel and more comprehensive reading keys for monuments' design can be achieved through a full utilization of previous studies and analytic approach based

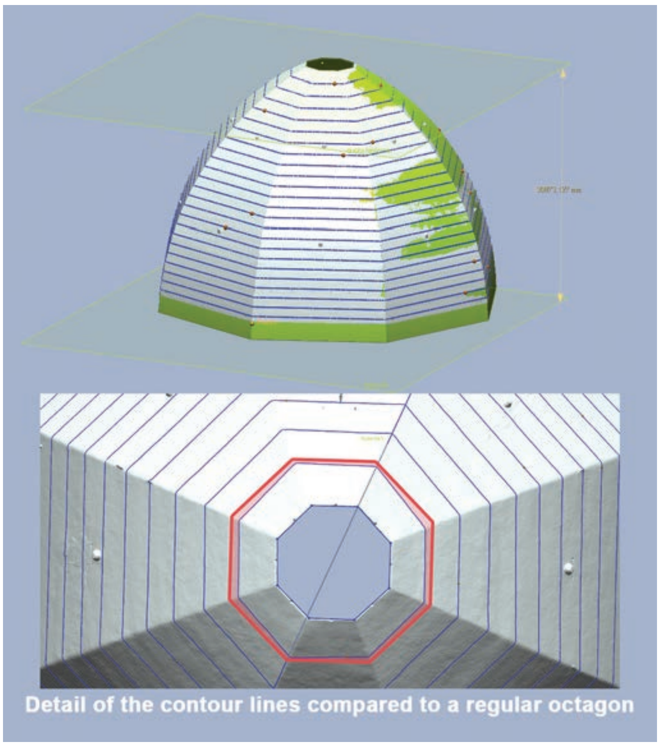


Fig. 10. Contour lines from horizontal reference planes and detail from the top of the irregular octagon at various levels in comparison to a regular one (in red).

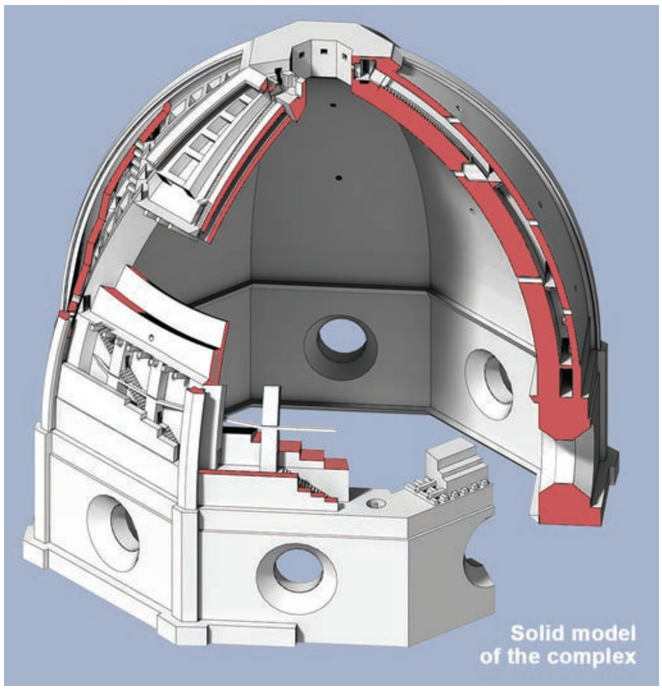


Fig. 11. Solid modelling of the whole Dome. Exploded axonometric view. Model by the author.

on geometrical and mathematical knowledge, such as current techniques from other fields.

In conclusion, it is evident that the updated technologies, as well as the latest devices, can both help the geometrical research with increased reliability, even on fully documented topics (fig. 10).

In the last analysis one can say that nowadays it is absolutely impossible, as well as methodologically mistaken, not to use up-to-date technologies for the in depth and critical research on architectural heritage and its proportional and metric dimensions.

Moreover it is important to point out the advantages in similar research to ascertain the accuracy achieved by a 3D reconstructive models and the accuracy of this model in comparison with the survey used for geometric analysis (Migliari 2004).

These digital models, as previously demonstrated, have a scientific purpose and they are not only a mere and optional virtual representation, owing to the encoding in the model itself of different fields of knowledge.

In this way it is also possible to generate conceptual solid models as final output of all these operations, showing the interior structure of the dome as well as the actual connections among the architectural elements and the whole complex (fig. 11).

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